SIMULATION OF THE DYNAMIC RESPONSE OF A SYNCHRONOUS GENERATOR

by

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DECLARATION

I hereby declare that this thesis is my own work and that it has not been submitted to arother university for a degree.

December 1990

RESUMé

This thesis uses the constructional detail of an alternator to derive the impedance matrix of the alternator. The 3-phase machine with a stationary reference frame is then transformed to a 2-phase machine with a rotating reference frame. This transformed impedance matrix is then used in the voltage equation of the machine to calculate the dynamic response. The thesis also discusses a method of relating a machine with more than two damper circuits to the standard textbook theory of a machine with only two damper circuits. Considerable space has been devoted to the implementation of the algorithms derived on a computer.

OPSOMMING

In hierdie tesis word 'n metode gegee waarin die konstruksionele data van 'n alternator gebruik word om die impedansie matriks van die alternator af te lei. Die drie-fasige masjien met 'n stilstaande verwysings raamwerk word dan getransformeer na 'n ekwivalente twee-fasige masjien met 'n sinchroon roterende werwysings raamwerk ten opsigte van die rotor. Hierdie getransformeerde impedansie matriks word dan gebruik in die spannings vergelyking van die masjien om die dinamiese gedrag te voorspel. 'n Metode word ook beskryf om 'n masjien met meer as twee demperbane te verwys na die standaard teksboek masjien met net twee demperbane. 'n Groot deel van die tesis is daaraan gewy om die afgeleide algoritmes op 'n rekenaar te implimenteer.

to Wendy

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CONTENTS

1. INTRODUCTION

In the beginning of the century A. Blondel created a new theory for synchronous machines by resolving the fundamental space component of MMF along the two axes of symmetry - the direct axis of the pole, and the quadrature axis between poles. Since then a lot has changed and the conventional machine theory has been replaced by the unified machine theory. Today, with the use of computers, finite element analysis gives an accurate representation of the practical machine with a small error between theoretical and practical values.

Finite element analysis unfortunately, takes quite a lot of time and sometimes a faster, although less accurate, solution, is needed. A solution based on the unified machine theory gives a fairly accurate answer in a much shorter time than a finite element analysis. In this thesis a way to combine the unified theory with modern technology is given. The basic transformations are used to set up the machine equations which are then used to simulate the machine on a computer.

In his book on the unified theory of machines, C.V. Jones [1] summarize the basic philosophy of the unified theory as follows: **"The performance of any machine under any conditions of operation may be analyzed or predetermined in terms of measurements made solely at the terminals and shaft of the machine when the machine is stationary."** In this thesis a computer program is used to calculate the parameters of the machine and these are then used to analyze the machine under different conditions of operation.

The transient theory of machines is mainly concerned with the determination of the variations of current, voltage or torque with time, due to a sudden change in supply and load conditions. Park [23] derived and developed a set of equations for current, voltage power and torque under steady and transient load conditions. His work is based on the work done by Blondel. Jones also used some of Park's equations to derive his set of equations and transformations.

A big problem in the examination of the dynamic behaviour of the synchronous machine, is the effect of the damper windings. The parameters of the damper windings can not be measured and can only be calculated from tests. Rankin [25] derived a circuit model for the representation of the damper bar circuits which included all the elements of the damper bar circuits. These equivalent circuits are, however, very complex and not easy to use in a simulation process. The IEEE working group [26] discussed in their paper the derivation of a number of models of different complexities for the rotor of the synchronous machine, as well as a way of obtaining data for the models. The most complex model is a model with three equivalent circuits in the q-axis and two equivalent damper circuits in the d-axis. The field circuit makes up the third circuit on the d-axis. The data for the models are either obtained from tests done on the machine under various conditions or from the manufacturers data. Dandeno et al. [17] did tests on different machines to validate different ways of obtaining data for the machine model. They used the most complex model described in [26]. From their tests they found that the standard model, based on the manufacturers data, is inadequate in simulating dynamic response. They further show that the frequency response techniques give a much superior performance.

A number of models for the synchronous machine have been developed by several investigators [29 - 32]. Most models are based on the variables obtained by Park's [23] equations. Subbarao et al. [11] give in their paper a discussion on the inter-relationships and transitions among the various mathematical models. Shackshaft [29] derived a general-purpose alternator model which include mechanical and electrical damping, iron saturation and saliency, and it permits the inclusion of voltage-regulator and governor action. The model, however, has not been developed for digital solution methods, only for analog computer applications. Humpage and Saha [31], developed a digital computer method for the dynamic response of a turbogenerator unit, based on Shackshaft's model. In the representation of the generator, a method is followed by which the effective inductance coefficients and time constants in the direct and quadrature axes operational equations, may be related to, or varied in accordance with, the magnetic conditions computed at each time step, thereby allowing for magnetic saturation. The eddy current paths in the solid rotor are first represented by equivalent short-circuited windings in each axis. An improved rotor representation is then developed from a model having a rectangular section and a smooth surface.

A number of authors [33 - 35] did a comparison between a number of models used for the simulation of generators. The general conclusion was that the effect of the damping circuits must be included into the model to obtain an accurate representation of the machine under all conditions. The method from which the data were obtained, also plays a major role in the accuracy of the model. All these methods, however, used data obtained from tests or from the designers data. In the thesis, a way will be shown to calculate the parameters of the machine from the constructional data and use these to calculate the dynamic response of the machine.

Shipley et al. [36] used the impedance matrix to produce a noniterative stability study of a power system. They used the Norton equivalent to eliminate the machine's internal generated voltage bus and thus reduces the size of the matrix. In this thesis the impedance matrix of the machine will be used to derive a real time simulation of the dynamic response of the machine. The impedance matrix is based on the two-axis theory as derived by Jones. In this thesis, saturation in the machine will be neglected. Ways to include saturation in the machine model, is described by Boldea and Nasar [12], as well as by Manning and Abdel Halim [20]. Boldea and Nasar adds fictitious windings to the rotor and stator to compensate for saturation. Manning and Abdel Halim use a method of updating the inductances and rotational voltage components on each time step according to the saturation of the machine. The inductance matrix is thus not a constant matrix anymore, but is dependent on the current matrix. This removes the error which is present when saturation is neglected, as in this thesis.

The thesis consists of three parts. The first part describes the transformation of the rotating three-phase machine to the stationary two-phase machine. The second part describes the use of the transformed impedance matrix for the determination of the dynamic response of the generator. The third part discusses a way of relating a machine with more than 2 damper circuits, to the conventional standard textbook theory of only one damper circuit on each axis.

2. DERIVATION OF TWO-PHASE IMPEDANCE MATRIX

The electrical machine can be viewed as an ordinary electrical circuit, and all the laws applying to electrical circuits must apply here as well. If the machine is looked at from an electric circuit point of view, a number of interconnected circuits are seen. These circuits form a number of voltage loops to which Kirchhoff's laws must apply. For the standard machine with two damper circuits as used in theoretical investigations, the following equation can be derived:

$$\begin{bmatrix} v_a \\ v_b \\ v_c \\ v_F \\ v_D \\ v_Q \end{bmatrix} = \begin{bmatrix} Z_{aa} & Z_{ab} & Z_{ac} & Z_{aF} & Z_{aD} & Z_{aQ} \\ Z_{ba} & Z_{bb} & Z_{bc} & Z_{bF} & Z_{bD} & Z_{bQ} \\ Z_{ca} & Z_{cb} & Z_{cc} & Z_{cF} & Z_{cD} & Z_{cQ} \\ Z_{Fa} & Z_{Fb} & Z_{Fc} & Z_{FF} & Z_{FD} & 0 \\ Z_{Da} & Z_{Db} & Z_{Dc} & Z_{DF} & Z_{DD} & 0 \\ Z_{Qa} & Z_{Qb} & Z_{Qc} & 0 & 0 & Z_{QQ} \end{bmatrix} * \begin{bmatrix} i_a \\ i_b \\ i_c \\ i_F \\ i_D \\ i_Q \end{bmatrix}$$

This equation describes the operating of the machine and must be satisfied under all conditions.

2.1 DERIVATION OF THREE-PHASE IMPEDANCE MATRIX

2.1.1 CONSTRUCTION OF MACHINE

The machine on which this thesis is based has the following construction data:

Number of poles:	6
Number of field windings:	155
Number of stator slots:	36
Coil sides per slot:	2
Turns per coil:	7

Coil span:	4 slots
Operating frequency:	400 Hz
Number of damper bars per pole:	5
Parallel circuits in field:	2
Parallel circuits in armature:	2

Figure 2.1 shows a section through the machine on which the stator slots as well as the rotor poles can be seen. In figure 2.2 the position of the damper bars and their corresponding circuits are shown.

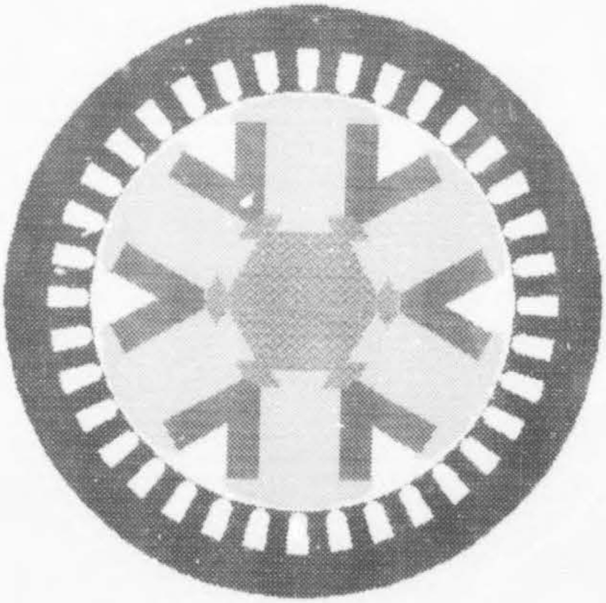

Figure 2.1: Section through machine showing stator slots and poles

There are 36 slots on the stator and thus 36 stator coils. On the rotor are six poles each with a field coil. On each pole are five damper bars which form two damper coils on the D-axis

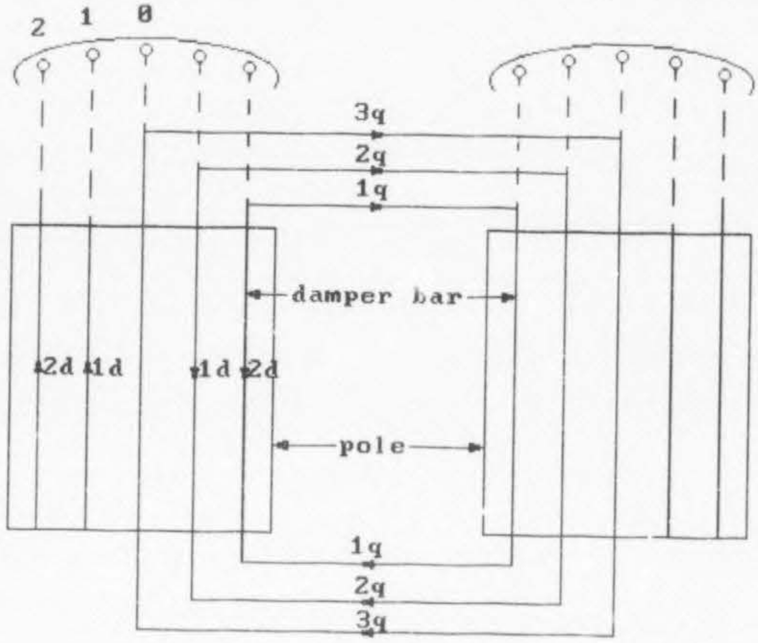

Figure 2.2: Damper bars with corresponding circuits

and three damper coils on the Q-axis of the machine. In total there are 72 coils in the machine, each with its own resistance and inductance. This impedance of a single coil and the mutual coupling between two coils will be discussed first.

2.1.2 CALCULATION OF INDUCTANCE

When a magnetic field varies with time, an electric field is produced in space. In the generator, the varying magnetic field in the stator core of the machine produces an induced voltage at the terminals of the machine, of a value determined by Faraday's law

$$e = N\frac{d\Phi}{dt} = \frac{d\lambda}{dt} \qquad 2.1.1$$

For a magnetic circuit with a linear relationship between ϕ and i, the λ-i relationship can be defined by the inductance as

$$L = \frac{\lambda}{i} \qquad 2.1.2$$

The inductance of a specific coil can be calculated for a specific rotor position θ if the flux linkage of the coil at that rotor position is known.

$$L(\theta) = \frac{\lambda(\theta)}{i} \qquad 2.1.3$$

The assumption is made initially that there is no saturation in the machine. There is thus a linear relationship between B and H. The way saturation is accounted for, is to calculate the level of saturation in the machine at a specific working point, and then change the impedance values by a factor according to the level of saturation. This forms part of an iterative loop that runs until the correct saturation level and parameter values for that working point is found.

To find the flux linkage of a coil, the flux density in the coil must be integrated over the area of the coil and multiplied by the number of turns in the coil. The distribution of the flux density in the coil must therefore be known. Consider the magnetic circuit shown in figure 2.3. The core is of uniform cross section and is excited by N windings carrying current i amperes (A). The MMF produced by this winding is equal to Ni. Thus

$$F = Ni \qquad 2.1.4$$

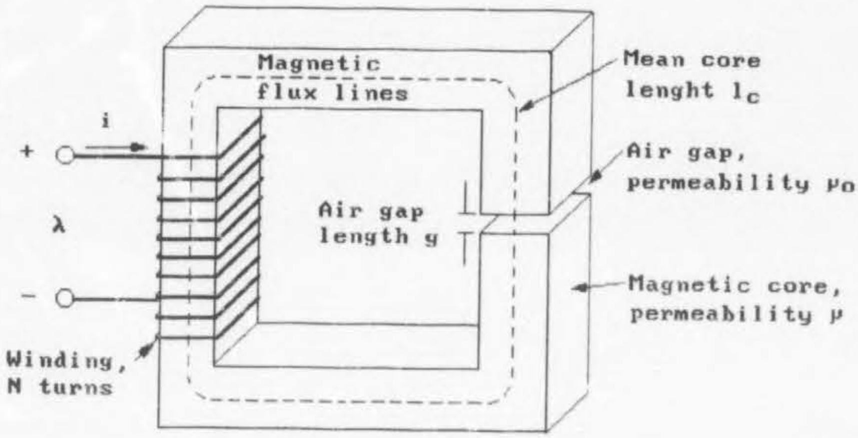

Figure 2.3: Magnetic circuit with air gap

The magnetic flux Φ crossing an area is the surface integral of the normal component of B; thus

$$\Phi = \int_S B \bullet da \qquad 2.1.5$$

The magnetic circuit shown in figure 2.3 can by analyzed as a magnetic circuit with two series components, a magnetic core with permeability μ and a mean length l_c, and an air gap with permeability μ_o and length g. Thus in the core

$$B_c = \frac{\Phi}{A_c} \qquad 2.1.6$$

and in the airgap

$$B_g = \frac{\Phi}{A_g} \qquad 2.1.7$$

The MMF applied to the circuit can be written as

$$\begin{aligned} F &= Ni \\ &= \frac{B_c}{\mu} l_c + \frac{B_g}{\mu_0} g \end{aligned} \qquad 2.1.8$$

From this we see that part of the MMF is required to excite the magnetic field of the core while the remainder excites the magnetic field in the air gap. From equations 2.1.6 and 7, equation 2.1.8 can be written in terms of the total flux Φ

$$F = \Phi \frac{l_c}{\mu A_c} + \Phi \frac{g}{\mu_0 A_g} \qquad 2.1.9$$

The terms which multiply with the flux in this equation are known as the reluctance, R, of the core and gap respectively,

$$R_c = \frac{l_c}{\mu A_c} \qquad 2.1.10$$

$$R_g = \frac{g}{\mu_0 A_g} \qquad 2.1.11$$

thus $$F = \Phi (R_c + R_g) \qquad 2.1.12$$

If saturation is neglected, the reluctance of the core is much lower than the reluctance of the airgap, due to the much higher permeability of the core. The flux and hence B can be found from equation 2.1.9 in terms of F and the air-gap properties alone.

$$\Phi \approx \frac{F}{R_g} = \frac{F\mu_0 A_g}{g} = Ni\frac{\mu_0 A_g}{g} \qquad 2.1.13$$

In the machine the values of the MMF and the length of the airgap varies over the pole arc. The value of the flux and hence B will thus vary with position in the airgap. In terms of position in the airgap, ϕ, the flux is

$$\begin{aligned}\Phi(\phi) &= N(\phi)\,\frac{\mu_0 A_g}{g(\phi)} \\ &= N(\phi)\,P_\phi(\phi)\,A_g\end{aligned} \qquad 2.1.14$$

where $P_\phi(\phi)$ is the permeance per unit area, abbreviated for convenience to permeance. Figure 2.4 shows a graph of the permeance, MMF and flux density distribution in the machine over two pole-pitches for one pole-phase group.

If there are two or more coils on the same magnetic axis, then there is a mutual coupling between the different coils. The self inductance of a coil can be found from the flux linkage of that coil due to its own current. The mutual inductance between two coils can be found from the flux linkage with say coil 1, due to the current in coil 2. From this, all the self and mutual inductances of the coils can be found. For the stator, it is easier to find the inductance of a pole-phase-group and not just a single coil. In the case of the machine under consideration, it means the inductance of two coils, the

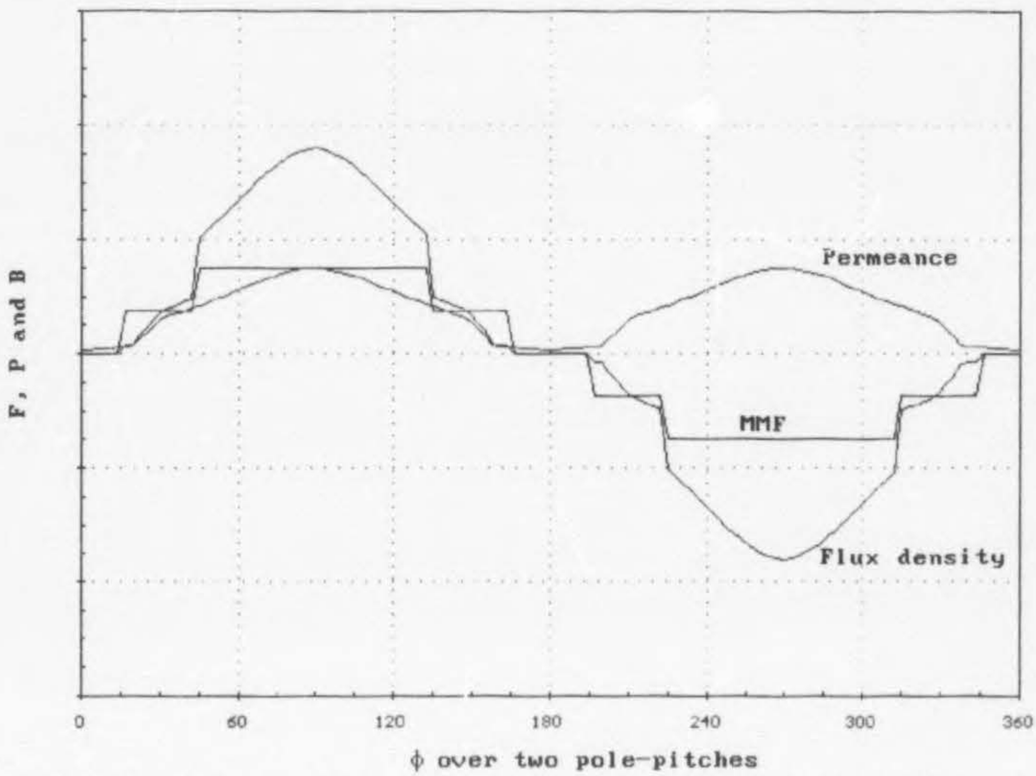

Figure 2.4: Permeance, MMF and flux density waveshapes as a function of ϕ

one displaced by one slot pitch from the other, in series. The resultant MMF of the two coils then includes the mutual coupling between the two coils, and it thus reduces the effective number of parameters on the stator to half the original number. Due to the effect that the machine is symmetrical, there are no coupling between different pole-phase groups nor between coils on separate poles.

Volschenk [2] gives in his thesis a detailed description of how the self and mutual inductances of the separate coils and pole-phase groups for this machine are calculated after the flux density distribution in the airgap was calculated. The MMF and permeance values in the airgap are calculated on a point by point basis as a function of ϕ. This ensures thus that harmonics are also accounted for, because the true shape of MMF and permeance are used to calculate the flux distribution.

2.1.3 CALCULATION OF RESISTANCES

The calculation of the resistances is done on the same way as described by Volschenk [2] in his thesis. The skin effect is ignored which means that the AC resistances are the same as the DC resistances.

2.1.4 INTERCONNECTION OF COILS

As mentioned, there are 36 coils on the stator, 6 field coils, 12 D-axis damper coils and 18 Q-axis damper coils. The stator coils are reduced to 18 due to the fact that calculation of the inductances are done on pole-phase-groups and not on single coils. This then reduces the total number of coils to 54. For the purpose of simplification in writing, the pole-phase groups on the stator will be look at as if each consists of only one coil.

The coils on the stator are divided into three phases, each phase having two parallel circuits. The field coils are also connected into two parallel circuits, each with three coils in series. For the damper coils, one must look at the currents flowing in the coils. Due to the symmetry of the machine, the currents in the D_1 coils on the different poles must be the same, because the flux generated in the poles are the same. The only difference is the direction of the current which changes from one pole to the next. This also apply to the rest of the damper coils.

Figures 2.5, 2.6 and 2.7 show the arrangement for the stator, field and D_1-damper circuits respectively. The rest of the damper circuits are connected in the same way as the D_1 circuit.

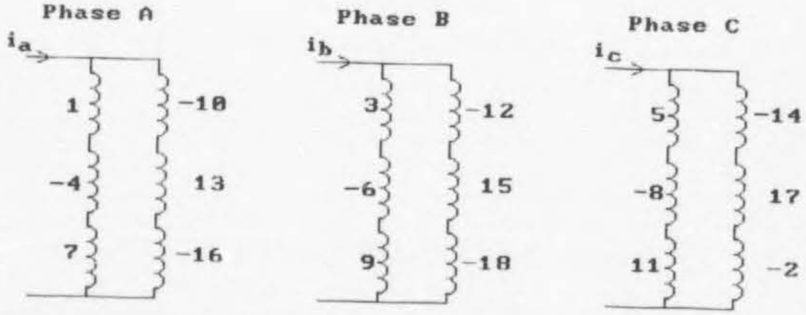

Figure 2.5: Interconnection of stator coils

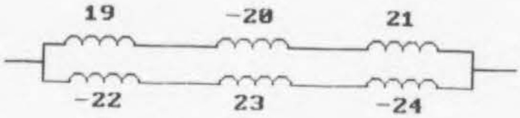

Figure 2.6: Interconnection of field coils

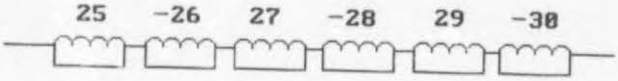

Figure 2.7: Interconnection of D_1-damper coils

2.1.5 DERIVATION OF COUPLING MATRIX

The impedance matrix of the machine must now be reduced from the current 54x54 matrix to a 9x9 matrix. The impedances of the different coils must be manipulated to find the impedance for each circuit.

From the figures in the previous paragraph, it can be seen that the same current flows through a number of coils. The 54 element current matrix I_{54} can thus be reduced to a 9 element matrix I with the use of a coupling matrix which can be derived from the figures in the previous paragraph. The relationship between the two matrices is

$$I_{54} = CI \qquad 2.1.15$$

The voltage equation in matrix form can be written as

$$V_{54} = Z_{54^2}I_{54} \qquad 2.1.16$$

where the subscibt **54** means a 54 element column matrix, and the subscribt **54²** means a 54x54 element square matrix.

This equation can be reduced to an equivalent 9 element equation V = ZI with the following transformations:

$$V = C^T V_{54}$$

$$Z = C^T Z_{54^2} C$$

$$I = C^T I_{54}$$

The coupling matrix will be given in three separate sections, one for each of the stator, field and damper coils, because the complete matrix is to big to fit onto a single page. On the left side of the matrix, the numbers of the original coils are given, while the subscripts for the equivalent circuits are given at the top of the matrix. By looking at the figures of the interconnection of the coils, the matrices can be derived with the following in mind. The current through a single pole-phase-group on the stator is equal to the stator phase-current divided by the number of parallel circuits which are two for the machine under consideration. The same is valid for the current through a single field coil.

All the currents in the stator section of the coupling matrix must be divided by the number of parallel circuits. It is therefore more convenient to put the factor in front of the matrix. The stator section is thus given by

$$C_S = \frac{1}{a_s} \cdot \begin{array}{c} \\ 1\\2\\3\\4\\5\\6\\7\\8\\9\\10\\11\\12\\13\\14\\15\\16\\17\\18 \end{array} \begin{array}{c} \begin{array}{ccccccccc} a & b & c & F & D_1 & D_2 & Q_1 & Q_2 & Q_3 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array} \qquad 2.1.17$$

The field currents must also be divided by the number of field parallel circuits and therefore also have a common factor in front of the matrix. The coupling matrix for the field is given by C_F.

$$C_F = \frac{1}{a_F} \cdot \begin{array}{c} \\ 19\\20\\21\\22\\23\\24 \end{array} \begin{array}{c} \begin{array}{ccccccccc} a & b & c & F & D_1 & D_2 & Q_1 & Q_2 & Q_3 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array} \qquad 2.1.18$$

The last section is the damper coils which includes 2 D-axis circuits and the 3 Q-axis circuits on each pole. This forms a 36x9 element matrix which is given by C_D.

$$C_D = \begin{array}{cc} & \begin{matrix} a & b & c & F & D_1 & D_2 & Q_1 & Q_2 & Q_3 \end{matrix} \\ \begin{matrix} 25\\26\\27\\28\\29\\30\\31\\32\\33\\34\\35\\36\\37\\38\\39\\40\\41\\42\\43\\44\\45\\46\\47\\48\\49\\59\\51\\52\\53\\54 \end{matrix} & \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix} \end{array} \qquad 2.1.19$$

2.1.6 REDUCTION OF IMPEDANCE MATRIX

From the coupling matrix and the relationship

$$Z = C^T Z_{54^2} C \qquad 2.1.20$$

the reduced impedance matrix can be found. The impedance matrix Z is a 9x9 element matrix. The expressions for the elements of Z can be found by performing the necessary matrix multiplications. It is found that the self-inductance of a phase winding on the stator is typically given by

$$L_a = \frac{p}{a_a^2} L_1 \qquad 2.1.21$$

where p is the number of poles and L_1 the self-inductance of a single pole-phase group. The mutual inductance between two phases is typically

$$M_{a,c} = \frac{p}{a_a^2} M_{1,2} \qquad 2.1.22$$

The self-inductance of the field winding is

$$L_F = \frac{p}{a_F^2} L_{19} \qquad 2.1.23$$

and the mutual coupling between the field and the stator

$$M_{a,F} = \frac{p}{a_a a_f} M_{1,19} \qquad 2.1.24$$

The self-inductance of an equivalent damper winding is

$$L_{D1} = pL_{25} \qquad 2.1.25$$

The mutual coupling between the damper winding and a stator winding is

$$M_{a,D_1} = \frac{p}{a_a} M_{1,25} \qquad 2.1.26$$

and the coupling between the damper winding and the field

$$M_{F,D_1} = \frac{p}{a_f} M_{19,25} \qquad 3.2.27$$

The coupling between two damper windings is

$$M_{D_1,D_2} = pM_{25,31} \qquad 2.1.28$$

This now forms the reduced 9x9 impedance matrix Z for the machine.

2.2 TRANSFORMATION TO 2-PHASE ROTATING REFERENCE FRAME

As mentioned earlier, the theory of the thesis is based on the unified machine theory. The keystone of the unified theory is the concept of transformations. Transformations used here are based on the direct and quadrature axis transformations developed by Park [23] to solve the problem of the synchronous machine.

The transformations are introduced with the purely mathematical object of simplifying and then solving the performance of rotating electrical machines. It is a process consisting of two transformations. The first transformation, the phase transformation C_1, replaces the actual three-phase machine with its two-phase equivalent. This simplifies the mathematical condition of the voltage equation considerably. The C_1 transformation is a dead transformation in the sense that its elements are all constants. Because C_1 is a dead transformation, it cannot eliminate the rotor angle θ which vary with time. The commutator transformation C_2, eliminates the rotor angle θ and removes the time dependent values. C_2 is a live transformation in the sense that its elements are functions of θ and hence of time.

For a transformation the relationship between the three-phase and the transformed values for the voltages, currents and impedances are given by the following equations:

$$
\begin{array}{ll}
I' = C^T I & I = CI' \\
V' = C^T & V = CV' \\
Z' = C^T Z C & Z = CZ'C^T
\end{array}
\qquad 2.2.1
$$

with V', I' and Z' the transformed matrices, C the transformation matrix and V, I and Z the matrices of the three-phase values.

For these relationships to be valid, the transformation matrix C must be orthogonal.

The mathematical derivation of the two transformation matrices is given in Appendices 2 and 3. The two transformation matrixes are

$$C_1 = \sqrt{\frac{2}{3}}\;\begin{array}{c} \\ r \\ y \\ b \end{array}\!\!\begin{array}{c} \begin{array}{ccc} 0 & \alpha & \beta \end{array} \\ \begin{bmatrix} \frac{1}{\sqrt{2}} & 1 & \\ \frac{1}{\sqrt{2}} & -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{1}{\sqrt{2}} & -\frac{1}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \end{array} \qquad 2.2.2$$

If the zero sequence component is ignored, the C_2-transformation reduces to

$$C_2 = \begin{array}{c} \\ \alpha \\ \beta \end{array}\!\!\begin{array}{c} \begin{array}{cc} q & d \end{array} \\ \begin{bmatrix} -\sin(\theta) & \cos(\theta) \\ \cos(\theta) & \sin(\theta) \end{bmatrix} \end{array} \qquad 2.2.3$$

The transformation of the impedance matrix will only be discussed superficially, as the mathematical process of the transformation is the same as described in Jones [1] chapters 15 and 16. The only difference is in the elements of the transformation matrices and the fact that the field system is on the rotor and not on the stator.

2.2.1 PHASE TRANSFORMATION

With the phase transformation the actual three-phase machine is replaced by its two-phase equivalent. The general voltage equation of the machine may be written in compound form as follows, with the suffixes R and S referring to the rotor and stator respectively.

$$\begin{bmatrix} v_R \\ v_S \end{bmatrix} = \begin{bmatrix} Z_{RR} & Z_{RS} \\ Z_{SR} & Z_{SS} \end{bmatrix} \cdot \begin{bmatrix} i_R \\ i_S \end{bmatrix} \qquad 2.2.4$$

The field system will be left unchanged because it is two-axis in nature. In compound form the transformation is therefore

$$C = \begin{bmatrix} 1 & \\ & C_1 \end{bmatrix} \qquad 2.2.5$$

and the transformed impedance matrix becomes

$$Z' = C_1^T Z C_1 = \begin{bmatrix} Z_{RR} & Z_{RS} C_1 \\ C_1^T Z_{SR} & C_1^T Z_{SS} C_1 \end{bmatrix} \qquad 2.2.6$$

2.2.2 COMMUTATOR TRANSFORMATION

The commutator transformation will only be applied to the stator voltages and currents and the voltage equation can once again be given by equation 2.2.4. The complete commutator transformation is therefore

$$C = \begin{bmatrix} 1 & \\ & C_2 \end{bmatrix} \qquad 2.2.7$$

The transformed impedance matrix therefore has the format

$$Z'' = C_2^T Z' C_2 = \begin{bmatrix} Z'_{RR} & Z'_{RS} C_2 \\ C_2^T Z'_{SR} & C_2^T Z'_{SS} C_2 \end{bmatrix} \qquad 2.2.8$$

The impedance matrix is now transformed to an impedance matrix of an equivalent two-phase machine with a reference frame rotating synchronously with the rotor. It is a 8x8 element matrix and is given in equation 2.2.9.

$$
Z'' = \begin{array}{cc}
 & \begin{array}{cccccccc} F & D_1 & D_2 & Q_1 & Q_2 & Q_3 & q & d \end{array} \\
\begin{array}{c} F \\ D_1 \\ D_2 \\ Q_1 \\ Q_2 \\ Q_3 \\ q \\ d \end{array} &
\left[\begin{array}{cccccccc}
R_F+L_F p & M_{FD_1}p & M_{FD_2}p & & & & & M_F p \\
M_{FD_1}p & R_{D_1}+L_{D_1}p & R_{D_1D_2}+M_{D_1D_2}p & & & & & M_{D_1}p \\
M_{FD_2}p & R_{D_1D_2}+M_{D_1D_2}p & R_{D_2}+L_{D_2}p & & & & & M_{D_2}p \\
 & & & R_{Q_1}+L_{Q_1}p & R_{Q_1Q_2}+M_{Q_1Q_2}p & R_{Q_1Q_3}+M_{Q_1Q_3}p & M_Q p & \\
 & & & R_{Q_1Q_2}+M_{Q_1Q_2}p & R_{Q_2}+L_{Q_2}p & R_{Q_2Q_3}+M_{Q_2Q_3}p & M_{Q_2}p & \\
 & & & R_{Q_1Q_3}+M_{Q_1Q_3}p & R_{Q_2Q_3}+M_{Q_2Q_3}p & R_{Q_3}+L_{Q_3}p & M_{Q_3}p & \\
-M_F\omega_r & -M_{D_1}\omega_r & -M_{D_2}\omega_r & M_{Q_1}p & M_{Q_2}p & M_{Q_3}p & R_S+L_q p & -L_d\omega_r \\
M_F p & M_{D_1}p & M_{D_2}p & M_{Q_1}\omega_r & M_{Q_2}\omega_r & M_{Q_3}\omega_r & L_q\omega_r & R_S+L_d p
\end{array}\right]
\end{array}
\qquad 2.2.9
$$

where p is the deritive operator d/dt.

The values of the elements are derived in the same way as described by Jones [1] and Volschenk [2]. The voltage equation can now be written as

$$V'' = Z'' I''$$

The rest of the work in this thesis is based on this voltage equation. When one look at the impedance matrix, it can be seen that the matrix includes speed terms, and the impedance matrix can thus be written as

$$Z'' = R + G + Lp$$

with R the resistance matrix, G the speed terms and L the inductance matrix.

3. DYNAMIC RESPONSE OF MACHINE

The response of a physical system to an applied excitation is determined by three factors: the input, the type of elements the system comprises of and how these elements are interconnected, and the history of the system. The input and the effects it produces in the system elements determine the steady-state response of the system if the input is constant or repetitive in time. The response of the system to a change in the input is described by the transient response. The response of the system to both a change in input and the initial conditions is called the complete response. In this chapter a way will be shown to derive the complete response of the machine after a change in condition. Saturation is neglected in the derivation of the dynamic response.

3.1 MATHEMATICAL MODEL OF MACHINE

Look at the RL-circuit in figure 3.1. These are the same type of elements one gets in the electrical machine.

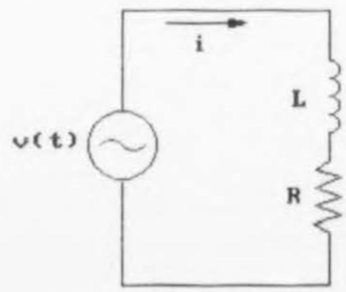

Figure 3.1: RL-circuit

The voltage equation for this circuit is

$$v(t) = L\frac{di(t)}{dt} + Ri(t) \tag{3.1.1}$$

The response of the current due to the voltage v(t) will be

$$\frac{di(t)}{dt} = \frac{v(t)}{L} - \frac{R}{L}i(t) \tag{3.1.2}$$

If the current i at a time $t = t_o$ is known, then the current at time $t = t_o+\Delta t$ can be calculated by integrating equation 3.1.2. On a computer this can be done quite easily with the use of numerical integration techniques.

The voltage equation used to simulate the performance of a generalised machine is given in matrix form by

$$[v] = [R][i] + \dot{\theta}[G][i] + p[X][i]$$

where the currents are the state variables. The [R], [G] and [X] matrices are the resistance, speed terms and inductance matrices respectively. The first and second terms of the above equation represent the resistive voltage and the rotational voltage components respectively, while the third term represents the transformer voltage component. The speed of the alternator is assumed to be constant, so the speed term is a constant which can be included into the G matrix.

3.2 NUMERICAL INTEGRATION

The current-equation for an electrical machine is a system of coupled first-order differential equations with initial conditions. With the use of numerical integration techniques the response of the current due to a change in load conditions can be calculated.

In practice there are a lot of methods to solve a system of coupled first-order differential equations, one of which is the Runge-Kutta method. The complete derivation of the mathematical formulation is given by Burden et al. [5] and will not be discussed here, but the basic structure of the method will be given. A more detailed explanation of the Runge-Kutta method is given in Appendix 4.

Given m first-order differential equations in the form

$$\begin{aligned} \frac{dx_1}{dt} &= f_1(t, x_1, x_2, \ldots, x_m) \\ \frac{dx_2}{dt} &= f_2(t, x_1, x_2, \ldots, x_m) \\ &\vdots \\ \frac{dx_m}{dt} &= f_m(t, x_1, x_2, \ldots, x_m) \end{aligned} \qquad 3.2.1$$

as well as the initial conditions for each x_j, the fourth order general Runge-Kutta method can be used to simultaneously get an approximate solution of the x_j's. The general Runge-Kutta formulas for these equations are given in the Appendix 4. The voltage equations of the machine must now be written in the same format as equation 3.2.1.

The voltage in matrix form can be rewritten as

$$[V] = [R][I] + [G][I] + [L]p[I] \qquad 3.2.2$$

where [R] the resistance matrix,
[G] the speed terms and
[L] the inductance matrix.

Saturation is neglected and the inductance terms are thus constants and not dependent on the currents. To get it into the format needed for the Runge-Kutta method, the equation must be written as follows:

$$\frac{d}{dt}[I] = [L]^{-1}[V] - [L]^{-1}([R] + [G])[I] \qquad 3.2.3$$

The Runge-Kutta method can now be used to approximate the values of the different currents at a time t+Δt if the values of the currents at time t are known.

3.3 IMPLEMENTATION OF ALGORITHM ON A COMPUTER

The Runge-Kutta algorithm can be programmed on a computer and used to calculate the dynamic response of the machine. In the previous paragraph it was shown how the machine equation can be written in a form from which the values of the currents can be calculated with numerical integration after a time Δt if the initial values are known. If this algorithm is done repeatedly with the most recent calculated values as the initial values of the next time-step, the response of the machine over a certain period of time after the change in load conditions, can be calculated.

The transformed 8x8 impedance matrix derived in chapter 2, is used for the calculations. To solve the voltage equation with the currents as the unknown quantities, one need to know the value of the voltages.

In the voltage matrix, the values of the damper voltages are zero, because all the damper circuits are shorted. Under short-circuit conditions, the terminal voltages of the machine are also zero and the field voltage is the only non-zero element in the matrix. The field voltage is supplied by an external source and can be taken as a constant voltage.

Under normal working conditions, the terminal voltages are not zero. If the machine is considered to be shorted, with the load impedance added as an external impedance to the armature impedance, then the machine is still under the same working conditions. The only difference is the position of the terminals. Normally the terminals are just on the outside of the machine, with the load connected to it. By looking at the load impedance as an external part of the machine, the terminals are now after the load and thus shorted for normal working conditions. It is only under no-load conditions that the terminals will not be shorted. By changing the value of the external impedance, different load conditions can be simulated. The field voltage is thus the only non-zero element in the voltage matrix under these conditions. Unfortunately, no-load can not be simulated this way. Theoretically, an infinitely large external impedance can be substituted as no-load. Experience showed that a load of about 4 - 5 % of full-load was the best achieved before the simulation diverged. This was thus taken as a no-load condition.

The initial values of the currents under steady-state conditions, are inputs from the keyboard. To ensure that the machine is in a steady-state condition, a simulation for the steady-state is done first, with the specified initial values of the currents. After a specified time, the load impedance is changed to simulate the new load condition. The last values of currents calculated are then used for the simulation of the dynamic response of the machine due to the change in load conditions. The whole simulation process can thus be divided into two parts, namely the steady-state simulation and the simulation of the dynamic response.

The Runge-Kutta algorithm can be explained according to the block diagram in figure 3.2. After the initialization, the time interval is divided by the number of points to get the time-step. The next step is to reset the counter. Then the four functions of the model [see Appendix 4] are calculated and summed to get the values of the currents at that time interval which is the starting time plus the time-step. The counter is then incremented and the time is shifted to the next time-step. This is repeated until the counter equals the number of points. After every time-step the values calculated is written to a file so that it can afterwards be displayed on a graph. An implementation of the algorithm in Turbo Pascal version 5 is shown in Appendix 5.

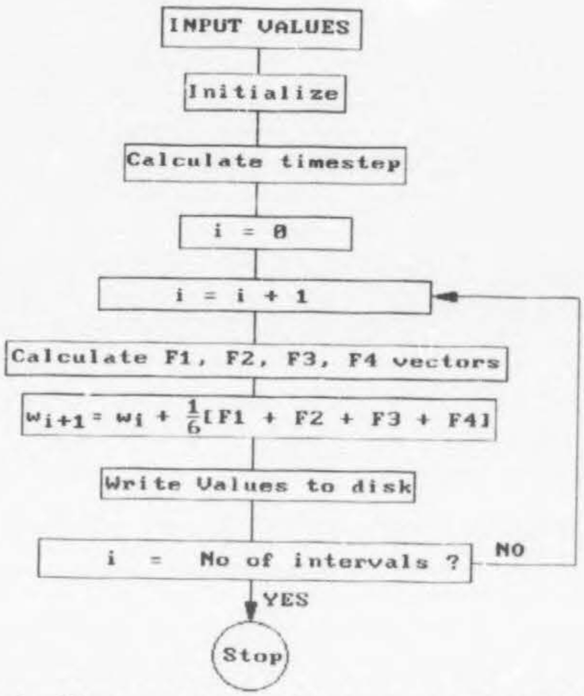

Figure 3.2: Block diagram of numerical integration procedure

The size of the time step plays an important role in the accuracy of the answer and must be made small to ensure greater accuracy. A too small time step also gives problems and makes the algorithm divergent. The reason for this is that in a too small time step, the round-off error becomes large enough to cause a considerable error in the answer. A detailed discussion on this is given by Burden et al. [5, chapter 6].

3.4 RESULTS

Different conditions of load and load change were simulated with the computer program. The results of the conditions are shown in the graphs in the following figures. Table 3.1 gives a summary of the different conditions simulated. The table shows the values of the load inductance and resistance for the steady-state as well as the values after a change in load condition.

NOTE: There are two separate scales for the armature and field currents, with the armature current on the left hand scale, and the field current on the right hand scale of the graphs.

Table 3.1: Impedance values for simulation of dynamic response

	Figure numbers	Before change		After change	
		R_1	L_1	R_1	L_1
No-load to full-load	3.2	50	0	2	0.7
Full-load to no-load	3.3	2	0.7	50	0
No-load to short-circuit	3.4/5	50	0	0	0
Short-circuit to no-load	3.6	0	0	50	0

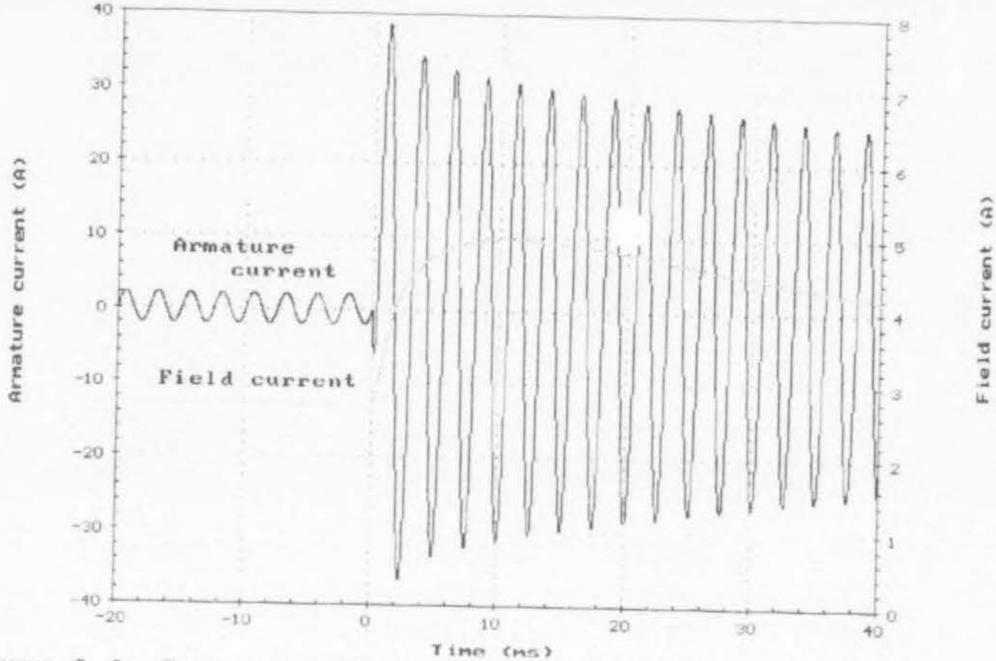

Figure 3.2: Response of currents: No-load to full-load

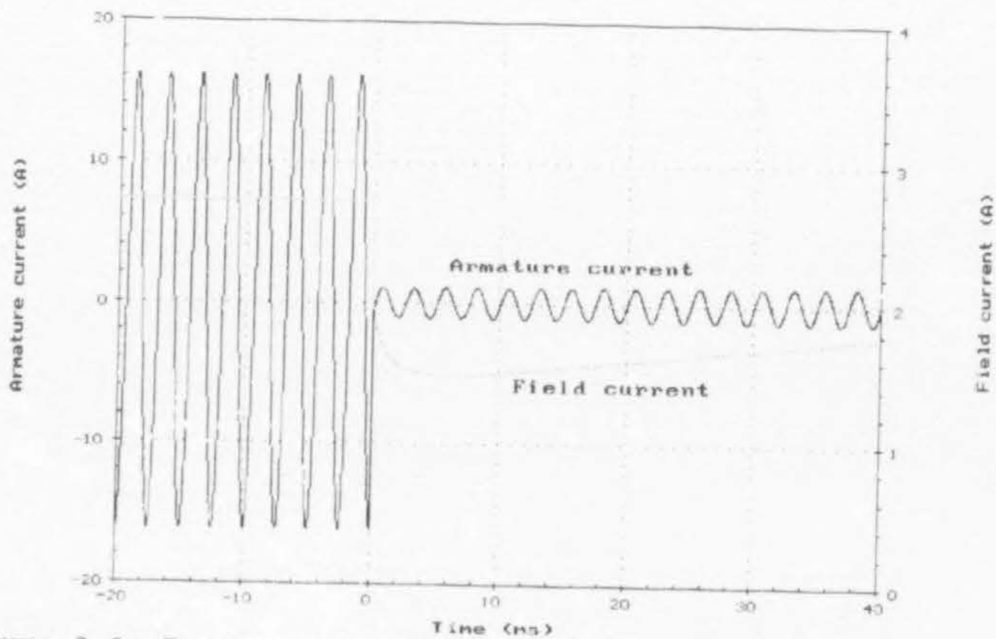

Figure 3.3: Response of currents: Full-load to no-load

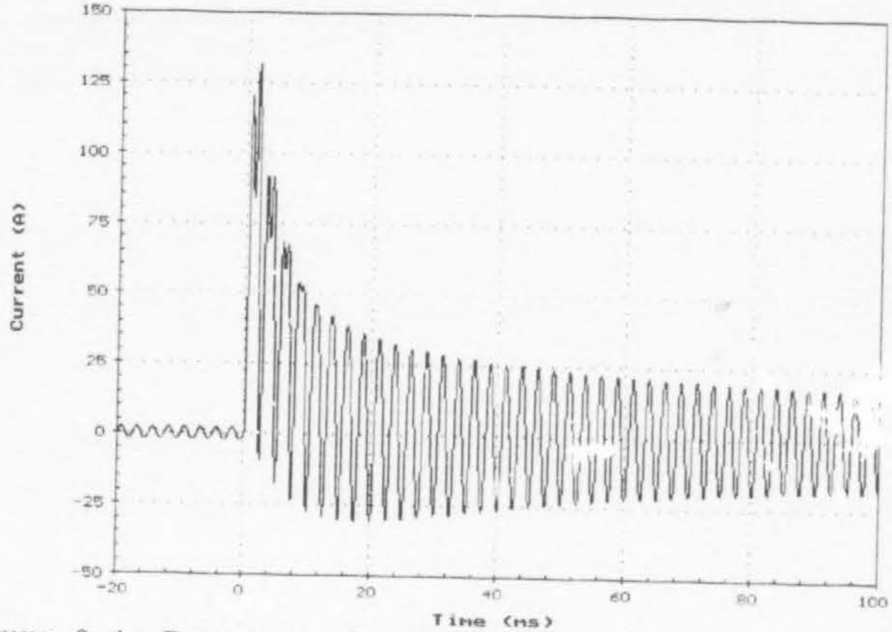

Figure 3.4: Response of armature current after short-circuit

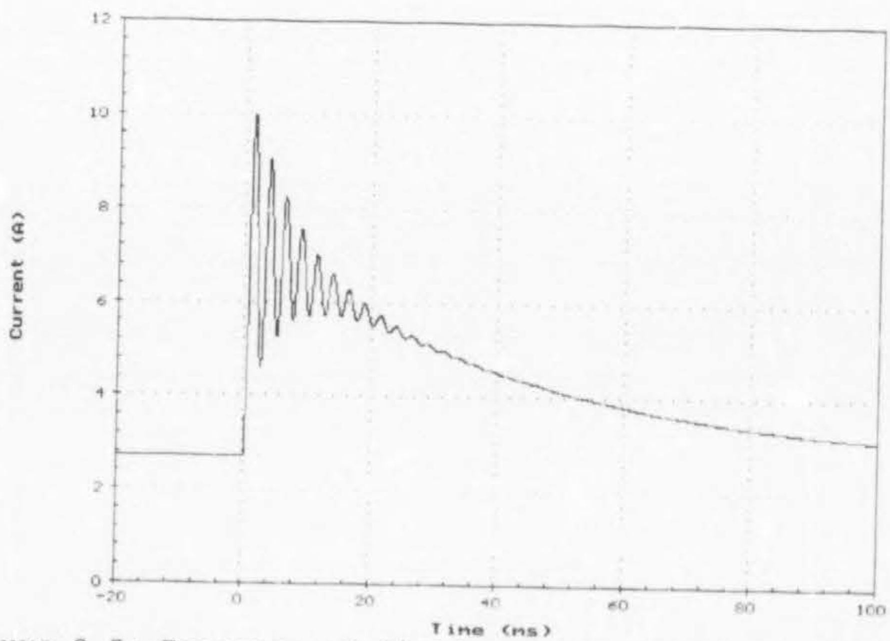

Figure 3.5: Response of field current after short-circuit

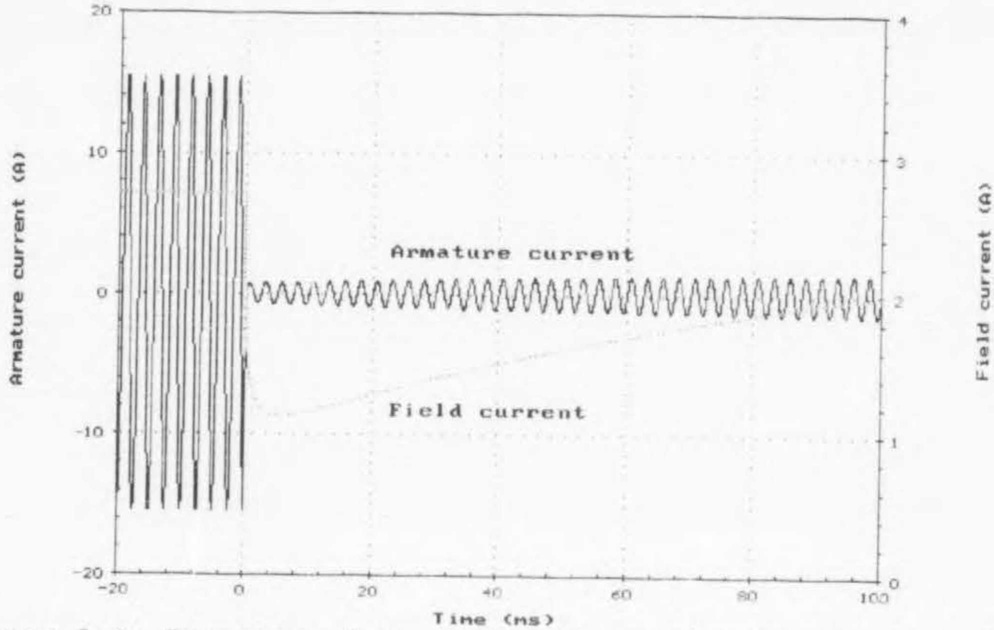

Figure 3.6: Response of currents: Short-circuit to no-load

3.5 CONCLUSIONS

These graphs display the simulated results of the machine for different changes in load conditions. Practical results of the machine is not available to verify the results, because it is a theoretical machine designed with the use of a computer program. The armature currents in the machine under no-load conditions is due to the external impedance added to the circuit. As explained earlier, no-load cannot be simulated, but only a load of about 4 - 5 % of full-load which is taken as no-load.

The rise in the field current from no-load to full-load, is due to the demagnetizing effect of the high armature current. From full-load to no-load, it is just the opposite. The quick drop in armature current cause over-magnetization of the machine and the field current drops to compensate for it. When a steady-state is reached, the field current is back to its normal value which depends on the values of the field resistance and the voltage applied. In figure 3.4 the effect of the damper bars can be seen in the first few cycles after the short-circuit. When a stator current appears suddenly, an opposing current is induced into the damper winding, forcing the stator flux into the airgap of the machine. This creates an effective leakage reactance which determines the initial high current value in the armature. With the exponential decay of the damper-winding current, the stator MMF is able to force its flux more deeply into the pole, against the opposition of the induced field current. Eventually the steady-state short-circuit condition is reached where the armature current is determined by the synchronous reactance. The ac component in the field current is due to the dc component in the armature windings, which vanishes as symmetry is approached.

4. REDUCTION OF NUMBER OF DAMPER CIRCUITS

The standard textbook unified machine theory addresses a machine with five windings, namely the stator d- and q-axis windings, the field winding, and the d- and q-axis damper windings. These are the basic windings for a synchronous machine. The machine discussed in this thesis however, has two d-axis and three q-axis damper windings on the rotor. To relate this machine to the standard textbook theory, the damper windings must be reduced to only two windings. The five damper windings must be replaced by two windings in such a way that the equivalent matrix still gives a fairly good representation of the original machine under all conditions. In this chapter, a way will be shown to find a mathematical 2-damper winding equivalent of the original five damper windings, as well as simulated results of the equivalent machine compared with the results of the original machine with 5 damper windings.

4.1 POSITION OF DAMPER BARS

The positions of the damper bars on the poles are shown in figure 4.1. Each damper winding stretches over a certain angle which is given in electrical degrees for the purpose of working out the flux linkages of the windings. In Table 4.1 the angle covered by each damper winding is shown according to the angles shown in figure 4.1.

Table 4.1: Position of damper bars on poles

Damper winding	Angle
D1	$\theta_3 \rightarrow \theta_5$
D2	$\theta_2 \rightarrow \theta_6$
Q1	$-\theta_2 \rightarrow \theta_2$
Q2	$-\theta_3 \rightarrow \theta_3$
Q3	$-\theta_4 \rightarrow \theta_4$

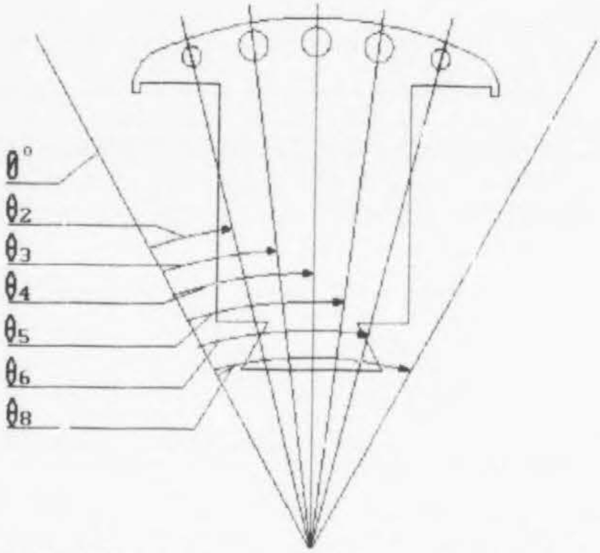

Figure 4.1: Sectional view of pole to show damper bar positions

The reduction of the d-axis damper windings will be easier than the q-axis windings and it will be used to find a method to reduce the number of windings.

4.2 REDUCTION OF D-AXIS DAMPER WINDINGS

One needs to find a mathematical equivalent of the original windings that will give a good representation of the characteristics of the original circuits. This means that the energy stored and the power dissipated in the equivalent winding must be the same as in the original windings.

Consider the two circuits shown in figure 4.2. They represent the two d-axis damper windings of the machine. Each circuit consists of an inductance, L, and a resistance, R. The inductance consists of a leakage component and a mutual component. The resistance consists of a mutual component as well as a separate component which together forms the total resistance of the circuits.

From figure 4.2 the following equations can be derived:

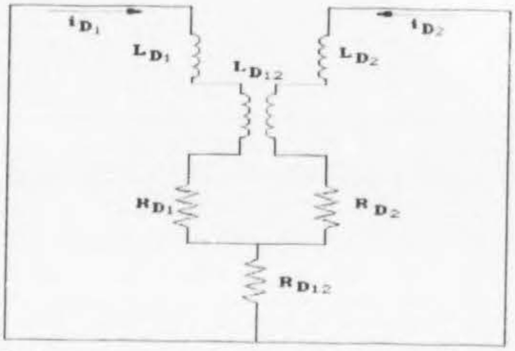

Figure 4.2: RL-circuit representation of d-axis damper circuit

$$L_{D_{11}} = L_{D_1} + L_{D_{12}}$$
$$L_{D_{22}} = L_{D_2} + L_{D_{12}}$$
$$R_{D_{11}} = R_{D_1} + R_{D_{12}}$$
$$R_{D_{22}} = R_{D_2} + R_{D_{12}}$$

In each of the circuits a voltage is induced due to currents flowing in the circuit itself as well as current flowing in the other circuit. From the relationships given above the equations for the induced voltages can be written as

$$e_{D_1} = L_{D_{11}} \frac{di_{D_1}}{dt} + L_{D_{12}} \frac{di_{D_2}}{dt}$$
$$e_{D_2} = L_{D_{21}} \frac{di_{D_1}}{dt} + L_{D_{22}} \frac{di_{D_2}}{dt} \qquad 4.2.1$$

In searching for a method to replace the 2 damper windings by 1 equivalent winding, it was found that a good estimation of dynamic response is obtained when it is assumed that the two damper currents have as an approximation the same waveshapes in time. The equivalent current must hence have the same waveshape in time and is taken as the sum of the two separate currents; thus

$$i_{D_e} = i_{D_1} + i_{D_2} \tag{4.2.2}$$

If it is assumed that all the currents have the same waveshape in time, then the two damper currents can be written as a constant factor of the equivalent current.

$$i_{D_1} = K_{D_1} i_{D_e} \tag{4.2.3}$$

$$i_{D_2} = K_{D_2} i_{D_e} \tag{4.2.4}$$

From these two equations and equation 4.2.2 the following can be derived:

$$\begin{aligned} K_{D_1} i_{D_e} + K_{D_2} i_{D_e} &= i_{D_e} \\ \therefore \quad K_{D_1} + K_{D_2} &= 1 \end{aligned} \tag{4.2.5}$$

This gives a relationship between the two current constants.

4.2.1 EQUIVALENT SELF INDUCTANCE

The energy stored in the equivalent circuit must have the same value as the total energy stored in the original circuits. From this, the energy stored in the equivalent circuit can be written as

$$\begin{aligned} W_{D_e} &= \tfrac{1}{2} L_{D_e} i_{D_e}^2 \\ &= W_{D_1} + W_{D_2} \end{aligned} \tag{4.2.6}$$

The energies stored in the separate windings are

$$
\begin{aligned}
W_{D_1} &= \tfrac{1}{2} L_{D_{11}} i_{D_1}^2 + \tfrac{1}{2} L_{D_{12}} i_{D_1} i_{D_2} \\
W_{D_2} &= \tfrac{1}{2} L_{D_{21}} i_{D_1} i_{D_2} + \tfrac{1}{2} L_{D_{22}} i_{D_2}^2
\end{aligned}
\qquad 4.2.7
$$

By substituting equations 4.2.3 and 4.2.4 into this equation one gets

$$
\begin{aligned}
W_{D_1} &= \tfrac{1}{2} L_{D_{11}} K_{D_1}^2 i_{D_e}^2 + \tfrac{1}{2} L_{D_{12}} K_{D_1} K_{D_2} i_{D_e}^2 \\
W_{D_2} &= \tfrac{1}{2} L_{D_{21}} K_{D_2} K_{D_1} i_{D_e}^2 + \tfrac{1}{2} L_{D_{22}} K_{D_2}^2 i_{D_e}^2
\end{aligned}
\qquad 4.2.8
$$

These two equations and equation 4.2.6 lead to

$$
\tfrac{1}{2} L_{D_e} i_{D_e}^2 = \tfrac{1}{2} K_{D_1}^2 L_{D_{11}} i_{D_e}^2 + \tfrac{1}{2} K_{D_2}^2 L_{D_{22}} i_{D_e}^2 + K_{D_1} K_{D_2} L_{D_{12}} i_{D_e}^2 \qquad 4.2.9
$$

from which the equivalent self inductance can be found as

$$
L_{D_e} = K_{D_1}^2 L_{D_{11}} + K_{D_2}^2 L_{D_{22}} + 2 K_{D_1} K_{D_2} L_{D_{12}} \qquad 4.2.10
$$

4.2.2 EQUIVALENT MUTUAL INDUCTANCES

To find the equivalent mutual inductances between the equivalent damper circuit and the stator or field, consider the equations for the voltages induced in the stator and field windings due to currents flowing in the damper circuits. The equations for the induced voltages are

$$e_{aD} = M_{aD_1} p i_{D_1} + M_{aD_2} p i_{D_2}$$
$$e_{FD} = M_{FD_1} p i_{D_1} + M_{FD_2} p i_{D_2} \qquad 4.2.11$$

Substituting the currents with equations 4.2.3 and 4 gives

$$e_{aD} = M_{aD_1} K_{D_1} p i_{D_e} + M_{aD_2} K_{D_2} p i_{D_e}$$
$$e_{FD} = M_{FD_1} K_{D_1} p i_{D_e} + M_{FD_2} K_{D_2} p i_{D_e} \qquad 4.2.12$$

The voltages induced due to an equivalent mutual inductance and current can be given as

$$e_{aD_e} = M_{aD_e} p i_{D_e}$$
$$e_{FD_e} = M_{FD_e} p i_{D_e} \qquad 4.2.13$$

From these two equations and the previous two, the equivalent mutual inductances are seen to be

$$M_{aD_e} = K_{D_1} M_{aD_1} + K_{D_2} M_{aD_2}$$
$$M_{FD_e} = K_{D_1} M_{FD_1} + K_{D_2} M_{FD_2} \qquad 4.2.14$$

4.2.3 EQUIVALENT RESISTANCE

The power dissipated in the equivalent circuit must have the same magnitude as the power dissipated in the original circuits. Thus

$$
\begin{aligned}
P_{D_e} &= R_{D_e} i_{D_e}^2 \\
&= P_{D_1} + P_{D_2}
\end{aligned}
\qquad 4.2.15
$$

The power dissipated in the separate windings are

$$
\begin{aligned}
P_{D_1} &= R_{D_{11}} i_{D_1}^2 + R_{D_{12}} i_{D_1} i_{D_2} \\
P_{D_2} &= R_{D_{12}} i_{D_1} i_{D_2} + R_{D_{22}} i_{D_2}^2
\end{aligned}
\qquad 4.2.16
$$

Substituting these two equations and equations 4.2.3 and 4 into equation 4.2.15 results in

$$
\begin{aligned}
R_{D_e} i_{D_e}^2 &= R_{D_{11}} i_{D_1}^2 + R_{D_{22}} i_{D_2}^2 + 2R_{D_{12}} i_{D_1} i_{D_2} \\
&= K_{D_1}^2 R_{D_{11}} i_{D_e}^2 + K_{D_2}^2 R_{D_{22}} i_{D_e}^2 + 2K_{D_1} K_{D_2} i_{D_e}^2
\end{aligned}
\qquad 4.2.17
$$

from which the equivalent resistance is found to be

$$
R_{D_e} = K_{D_1}^2 R_{D_{11}} + K_{D_2}^2 R_{D_{22}} + 2K_{D_1} K_{D_2} R_{D_{12}} \qquad 4.2.18
$$

4.2.4 CALCULATING CURRENT CONSTANTS

If the constants K_{D1} and K_{D2} are known, the equivalent parameters can be calculated.

To calculate the constants K_{D1} and K_{D2} look at the equations for the induced voltages again. These voltages can also be written in terms of the flux linkages of the coils as follows.

$$e_{D_1} = \frac{d\lambda_{D_1}}{dt} = \frac{d\lambda_{D_{11}}}{dt} + \frac{d\lambda_{D_{12}}}{dt}$$
$$e_{D_2} = \frac{d\lambda_{D_2}}{dt} = \frac{d\lambda_{D_{21}}}{dt} + \frac{d\lambda_{D_{22}}}{dt} \qquad 4.2.19$$

The flux linkage of a coil can be calculated by integrating the flux density over the area of the coil, which gives then the flux linkage of a single turn. Multiplying this with the number of turns gives the total number of flux linkages by the coil. If the distribution of the flux density in the airgap is known, it can be integrated as a function of the angle in the airgap. The following set of equations gives a general method of calculating the flux linkage of a coil.

$$\lambda = N\int_{\psi_1}^{\psi_2} B(\phi)\, rL_{axial} d\phi$$
$$= NrL_{axial}\int_{\psi_1}^{\psi_2} FP_{\phi}(\phi)\, d\phi$$
$$= NrL_{axial}\int_{\psi_1}^{\psi_2} NiP_{\phi}(\phi)\, d\phi \qquad 4.2.20$$
$$= N^2 rL_{axial} i\int_{\psi_1}^{\psi_2} P_{\phi}(\phi)\, d\phi$$
$$\therefore \frac{d\lambda}{dt} = N^2 rL_{axial}\int_{\psi_1}^{\psi_2} P_{\phi}(\phi)\, d\phi \;\frac{di}{dt}$$

where N number of turns
r radius of machine
L_{axial} axial length of machine
$B(\phi)$ flux density distribution in airgap
$P_{\phi}(\phi)$ permeance per unit area
ψ_1, ψ_2 start and end points of angle covered by coil

For the damper windings the number of turns is one and the radius times the axial length is a constant which can be taken as C for the time being. The start and end points of the angle covered by the coil can be found from table 4.1. The induced voltages in the two circuits can hence be written as

$$e_{D_1} = C \cdot \left(K_{D_1} \int_{\theta_3}^{\theta_5} P_\phi(\phi)\, d\phi + K_{D_2} \int_{\theta_3}^{\theta_5} P_\phi(\phi)\, d\phi \right) \cdot \frac{di_{D_e}}{dt}$$

$$e_{D_2} = C \cdot \left(K_{D_1} \int_{\theta_3}^{\theta_5} P_\phi(\phi)\, d\phi + K_{D_2} \int_{\theta_2}^{\theta_6} P_\phi(\phi)\, d\phi \right) \cdot \frac{di_{D_e}}{dt} \qquad 4.2.21$$

From these equations one can find the relationship between e_{D1} and e_{D2}. It is seen that their ratio is a constant, K_{VF}. The only value in the equations that can vary, is the current, and it is not present in the ratio. From equation 4.2.21 this constant is

$$K_{VF} = \frac{e_{D_1}}{e_{D_2}} = \frac{\left(K_{D_1} \int_{\theta_3}^{\theta_5} P_\phi(\phi)\, d\phi + K_{D_2} \int_{\theta_3}^{\theta_5} P_\phi(\phi)\, d\phi \right)}{\left(K_{D_1} \int_{\theta_3}^{\theta_5} P_\phi(\phi)\, d\phi + K_{D_2} \int_{\theta_2}^{\theta_6} P_\phi(\phi)\, d\phi \right)} \qquad 4.2.22$$

This constant gives a relationship between the voltages induced in the two circuits. From this relationship and equation 4.2.1 the following equation is found:

$$K_{D_1}L_{D_{11}}\frac{di_{D_e}}{dt} + K_{D_2}L_{D_{12}}\frac{di_{D_e}}{dt} = K_{VF}(K_{D_1}L_{D_{12}}\frac{di_{D_e}}{dt} + K_{D_2}L_{D_{22}}\frac{di_{D_e}}{dt})$$

4.2.23

By eliminating the di/dt term on both sides of the equation, it reduces to

$$K_{D_1}L_{D_{11}} + K_{D_2}L_{D_{12}} = K_{VF}*(K_{D_1}L_{D_{12}} + K_{D_2}L_{D_{22}}) \qquad 4.2.24$$

In this equation the constant term K_{VF} can be written in terms of K_{D1} and K_{D2}. This equation together with equation 4.2.5 thus gives two equations with two unknown quantities. The values of the two constants can hence be found. These values of the constants are then substitute back into equations 4.2.10, 14 and 18 to find the values of the equivalent parameters of self and mutual inductance as well as resistance.

4.3 Q-AXIS WINDINGS

The reduction will be done in the same way as with the D-axis, but the number of circuits are now three instead of only two. This makes the mathematics much more complicated than with the D-axis.

As with the D-axis currents, the assumption is again made that the three currents have the same waveshape in time. The equivalent current must hence have the same waveshape in time and is taken as the sum of the three currents. Thus

$$i_{Q_e} = i_{Q_1} + i_{Q_2} + i_{Q_3} \qquad 4.3.1$$

Form the fact that the currents have the same waveshape, the following relationships can be derived.

$$i_{Q_1} = K_{Q_1} i_{Q_e} \qquad 4.3.2$$

$$i_{Q_2} = K_{Q_2} i_{Q_e} \qquad 4.3.3$$

$$i_{Q_3} = K_{Q_3} i_{Q_e} \qquad 4.3.4$$

From equations 4.3.1 - 4 one can derive the following:

$$K_{Q_1} + K_{Q_2} + K_{Q_3} = 1 \qquad 4.3.5$$

The equations for the voltages induced in the three circuits are

$$\begin{aligned} e_{Q_1} &= L_{Q_{11}}\frac{di_{Q_1}}{dt} + L_{Q_{12}}\frac{di_{Q_2}}{dt} + L_{Q_{13}}\frac{di_{Q_3}}{dt} \\ e_{Q_2} &= L_{Q_{21}}\frac{di_{Q_1}}{dt} + L_{Q_{22}}\frac{di_{Q_2}}{dt} + L_{Q_{23}}\frac{di_{Q_3}}{dt} \\ e_{Q_3} &= L_{Q_{31}}\frac{di_{Q_1}}{dt} + L_{Q_{32}}\frac{di_{Q_2}}{dt} + L_{Q_{33}}\frac{di_{Q_3}}{dt} \end{aligned} \qquad 4.3.6$$

By substituting the currents with the relationships in equations 4.3.2 - 4, equation 4.3.6 now becomes

$$\begin{aligned} e_{Q_1} &= K_{Q_1}L_{Q_{11}}\frac{di_{Q_e}}{dt} + K_{Q_2}L_{Q_{12}}\frac{di_{Q_e}}{dt} + K_{Q_3}L_{Q_{13}}\frac{di_{Q_e}}{dt} \\ e_{Q_2} &= K_{Q_1}L_{Q_{21}}\frac{di_{Q_e}}{dt} + K_{Q_2}L_{Q_{22}}\frac{di_{Q_e}}{dt} + K_{Q_3}L_{Q_{23}}\frac{di_{Q_e}}{dt} \\ e_{Q_3} &= K_{Q_1}L_{Q_{31}}\frac{di_{Q_e}}{dt} + K_{Q_2}L_{Q_{32}}\frac{di_{Q_e}}{dt} + K_{Q_3}L_{Q_{33}}\frac{di_{Q_e}}{dt} \end{aligned} \qquad 4.3.7$$

4.3.1 EQUIVALENT SELF INDUCTANCE

The same concept as used with the d-axis will be used here. The energy stored in the equivalent impedance must be the same as the energy stored in the original impedances. The energy stored in the equivalent circuits can hence be written as

$$\begin{aligned} W_{Q_T} &= W_{Q_1} + W_{Q_2} + W_{Q_3} \\ &= \tfrac{1}{2} L_{Q_e} i_{Q_e}^2 \end{aligned} \qquad 4.3.8$$

The energies stored in the separate windings are

$$\begin{aligned} W_{Q_1} &= \tfrac{1}{2}[L_{Q_{11}} i_{Q_1}^2 + L_{Q_{12}} i_{Q_1} i_{Q_2} + L_{Q_{13}} i_{Q_1} i_{Q_3}] \\ W_{Q_2} &= \tfrac{1}{2}[L_{Q_{12}} i_{Q_1} i_{Q_2} + L_{Q_{22}} i_{Q_2}^2 + L_{Q_{23}} i_{Q_2} i_{Q_3}] \\ W_{Q_3} &= \tfrac{1}{2}[L_{Q_{13}} i_{Q_1} i_{Q_3} + L_{Q_{23}} i_{Q_2} i_{Q_3} + L_{Q_{33}} i_{Q_3}^2] \end{aligned} \qquad 4.3.9$$

By substituting the currents with equations 4.3.2 - 4, one gets

$$\begin{aligned} W_{Q_1} &= \tfrac{1}{2}[K_{Q_1}^2 L_{Q_{11}} + K_{Q_1} K_{Q_2} L_{Q_{12}} + K_{Q_1} K_{Q_3} L_{Q_{13}}]\, i_{Q_e}^2 \\ W_{Q_2} &= \tfrac{1}{2}[K_{Q_1} K_{Q_2} L_{Q_{12}} + K_{Q_2}^2 L_{Q_{22}} + K_{Q_2} K_{Q_3} L_{Q_{23}}]\, i_{Q_e}^2 \\ W_{Q_3} &= \tfrac{1}{2}[K_{Q_1} K_{Q_3} L_{Q_{13}} + K_{Q_2} K_{Q_3} L_{Q_{23}} + K_{Q_3}^2 L_{Q_{33}}]\, i_{Q_e}^2 \end{aligned} \qquad 4.3.10$$

Substituting these equations into equation 4.3.8 leads to

$$\tfrac{1}{2} L_{Q_e} i_{Q_e}^2 = \left[\tfrac{1}{2}\left(K_{Q_1}^2 L_{Q_{11}} + K_{Q_2}^2 L_{Q_{22}} + K_{Q_3}^2 L_{Q_{33}}\right) + K_{Q_1} K_{Q_2} L_{Q_{12}} + K_{Q_1} K_{Q_3} L_{Q_{13}} + K_{Q_2} K_{Q_3} L_{Q_{23}}\right] i_{Q_e}^2 \qquad 4.3.11$$

from which the equivalent self inductance is found to be

$$L_{Q_e} = K_{Q_1}^2 L_{Q_{11}} + K_{Q_2}^2 L_{Q_{22}} + K_{Q_3}^2 L_{Q_{33}} + 2K_{Q_1}K_{Q_2}L_{Q_{12}} + 2K_{Q_1}K_{Q_3}L_{Q_{13}} + 2K_{Q_2}K_{Q_3}L_{Q_{23}} \quad 4.3.12$$

4.3.2 EQUIVALENT MUTUAL INDUCTANCES

The q-axis damper windings have only mutual coupling with the q-axis stator windings. As with the d-axis windings, consider the voltages induced in the stator windings due to currents flowing in the damper windings. The induced voltage is

$$e_{qQ} = M_{qQ_1} p i_{Q_1} + M_{qQ_2} p i_{Q_2} + M_{qQ_3} p i_{Q_3} \quad 4.3.13$$

Substituting the currents with equations 4.3.2 - 4 gives

$$e_{qQ} = \left[M_{qQ_1}K_{Q_1} + M_{qQ_2}K_{Q_2} + M_{qQ_3}K_{Q_3}\right] p i_{Q_e} \quad 4.3.14$$

The voltage induced in the stator due to an equivalent mutual inductance and current can be written as

$$e_{qQ_e} = M_{qQ_e} p i_{Q_e} \quad 4.3.15$$

From this equation and equation 4.3.14 the equivalent mutual inductance is found to be

$$M_{qQ_e} = K_{Q_1}M_{qQ_1} + K_{Q_2}M_{qQ_2} + K_{Q_3}M_{qQ_3} \quad 4.3.16$$

4.3.3 EQUIVALENT RESISTANCE

As before, the power dissipated in the equivalent resistance must be the same as the total power dissipated in the original resistances. Thus

$$\begin{aligned} P_{Q_e} &= R_{Q_e} i_{Q_e}^2 \\ &= P_{Q_1} + P_{Q_2} + P_{Q_3} \end{aligned} \qquad 4.3.17$$

The power dissipated in the three separate windings are

$$\begin{aligned} P_{Q_1} &= i_{Q_1}^2 R_{Q_{11}} + i_{Q_1} i_{Q_2} R_{Q_{12}} + i_{Q_1} i_{Q_3} R_{Q_{13}} \\ P_{Q_2} &= i_{Q_2} i_{Q_1} R_{Q_{21}} + i_{Q_2}^2 R_{Q_{22}} + i_{Q_2} i_{Q_3} R_{Q_{23}} \\ P_{Q_3} &= i_{Q_3} i_{Q_1} R_{Q_{31}} + i_{Q_3} i_{Q_2} R_{Q_{32}} + i_{Q_3}^2 R_{Q_{33}} \end{aligned} \qquad 4.3.18$$

By substituting the currents in these equations with equations 4.3.2 - 4, the power dissipated in the equivalent circuit can now be written as

$$\begin{aligned} P_{Q_T} &= i_{Q_e}^2 R_{Q_e} \\ &= \left[K_{Q_1}^2 R_{Q_{11}} + K_{Q_2}^2 R_{Q_{22}} + K_{Q_3}^2 R_{Q_{33}} + 2\left(K_{Q_1} K_{Q_2} R_{Q_{12}} + K_{Q_1} K_{Q_3} R_{Q_{13}} + K_{Q_2} K_{Q_3} R_{Q_{23}}\right)\right] i_{Q_e}^2 \end{aligned}$$

$$4.3.19$$

from which the equivalent resistance is found to be

$$R_{Q_e} = K_{Q_1}^2 R_{Q_{11}} + K_{Q_2}^2 R_{Q_{22}} + K_{Q_3}^2 R_{Q_{33}} + 2K_{Q_1} K_{Q_2} R_{Q_{12}} + 2K_{Q_1} K_{Q_3} R_{Q_{13}} + 2K_{Q_2} K_{Q_3} R_{Q_{23}}$$

$$4.3.20$$

The next step is to calculate the values of the three constants, from which the equivalent impedance values can then be found.

4.3.4 CALCULATING CURRENT CONSTANTS

To calculate the current constants, look at the equations for the induced voltages again. These voltages in terms of the flux linkages are

$$
\begin{aligned}
e_{Q_1} &= \frac{d\lambda_{Q_1}}{dt} = \frac{d\lambda_{Q_{11}}}{dt} + \frac{d\lambda_{Q_{12}}}{dt} + \frac{d\lambda_{Q_{13}}}{dt} \\
e_{Q_2} &= \frac{d\lambda_{Q_2}}{dt} = \frac{d\lambda_{Q_{21}}}{dt} + \frac{d\lambda_{Q_{22}}}{dt} + \frac{d\lambda_{Q_{23}}}{dt} \qquad 4.3.21 \\
e_{Q_3} &= \frac{d\lambda_{Q_3}}{dt} = \frac{d\lambda_{Q_{31}}}{dt} + \frac{d\lambda_{Q_{32}}}{dt} + \frac{d\lambda_{Q_{33}}}{dt}
\end{aligned}
$$

The flux linkages are calculated in the same way as described in paragraph 4.2.4 with the d-axis damper windings. The angles covered by a half of each coil are found in Table 4.1. The induced voltages can thus be written as

$$e_{Q_1} = 2\left(K_{Q_1}\int_{\Theta_0}^{\Theta_2} P_\phi d\phi + K_{Q_2}\int_{\Theta_0}^{\Theta_2} P_\phi d\phi + K_{Q_3}\int_{\Theta_0}^{\Theta_2} P_\phi d\phi\right)\frac{di_{Q_e}}{dt} \qquad 4.3.22$$

$$e_{Q_2} = 2\left(K_{Q_1}\int_{\Theta_0}^{\Theta_2} P_\phi d\phi + K_{Q_2}\int_{\Theta_0}^{\Theta_3} P_\phi d\phi + K_{Q_3}\int_{\Theta_0}^{\Theta_3} P_\phi d\phi\right)\frac{di_{Q_e}}{dt} \qquad 4.3.23$$

$$e_{Q_3} = 2\left(K_{Q_1}\int_{\Theta_0}^{\Theta_2} P_\phi d\phi + K_{Q_2}\int_{\Theta_0}^{\Theta_3} P_\phi d\phi + K_{Q_3}\int_{\Theta_0}^{\Theta_4} P_\phi d\phi\right)\frac{di_{Q_e}}{dt} \qquad 4.3.24$$

The following substitutions are made to simplify the mathematics:

$$F_{Q_1} = \int_{\theta_0}^{\theta_2} P_\phi d\phi \qquad 4.3.25$$

$$F_{Q_2} = \int_{\theta_0}^{\theta_3} P_\phi d\phi \qquad 4.3.26$$

$$F_{Q_3} = \int_{\theta_0}^{\theta_4} P_\phi d\phi \qquad 4.3.27$$

From these equations and equations 4.3.22 - 24 the following two equations can be derived which give a relationship between the voltages induced in the circuits.

$$\frac{e_{Q_1}}{e_{Q_2}} = \frac{F_{Q_1}}{K_{Q_1}F_{Q_1} + K_{Q_2}F_{Q_2} + K_{Q_3}F_{Q_2}} = F_{Q_{12}} \qquad 4.3.28$$

$$\frac{e_{Q_1}}{e_{Q_3}} = \frac{F_{Q_1}}{K_{Q_1}F_{Q_1} + K_{Q_2}F_{Q_2} + K_{Q_3}F_{Q_3}} = F_{Q_{13}} \qquad 4.3.29$$

Relate the voltage equations in equation 4.3.7 to each other according to equations 4.3.28 and 29 to get the following two equations.

$$L_{Q_{11}}K_{Q_1} + L_{Q_{12}}K_{Q_2} + L_{Q_{13}}K_{Q_3} = F_{Q_{12}} * (L_{Q_{12}}K_{Q_1} + L_{Q_{22}}K_{Q_2} + L_{Q_{23}}K_{Q_3}) \qquad 4.3.30$$

$$L_{Q_{11}}K_{Q_1} + L_{Q_{12}}K_{Q_2} + L_{Q_{13}}K_{Q_3} = F_{Q_{13}} * (L_{Q_{13}}K_{Q_1} + L_{Q_{23}}K_{Q_2} + L_{Q_{33}}K_{Q_3}) \qquad 4.3.31$$

These two equations together with equation 4.3.5 give three equations with three unknown values. From this the values of the three constants can now be calculated from which the equivalent self and mutual impedance as well as the resistance of the Q-axis can be calculated.

4.4 RESULTS OF EQUIVALENT CIRCUITS

The response of the damper currents after a sudden short-circuit, are displayed in the graphs in figures 4.3 and 4.4. Figure 4.3 display the response of the d-axis damper circuit currents and figure 4.4 the response of the q-axis damper circuit currents. From the graph it can be seen that the currents do not the have the same waveshapes as assumed. The difference in shape is the most significant for the sub-transient part just after the short-circuit. After a while, the waveshapes are quite similar. This would hence mean that the response of the reduced matrix would differ in the sub-transient part, but should be fairly accurate for the transient part of the dynamic response of the machine.

The original damper circuits are replaced by the mathematical equivalent to form a five by five impedance matrix. To verify the accuracy of the method, the dynamic response of the machine is calculated with the same program as before, but with the reduced impedance matrix. The results are then compared with the results of the original machine. In the following graphs the results for two different machines are shown. The second machine is an experimental machine with a dynamic response that differs from the first machine. This gives more value to the verification process.

The waveforms calculated with the reduced matrix as well as with the original matrix are displayed on the same graph. The first two graphs are the standard generator and the next two are the graphs of the second generator.

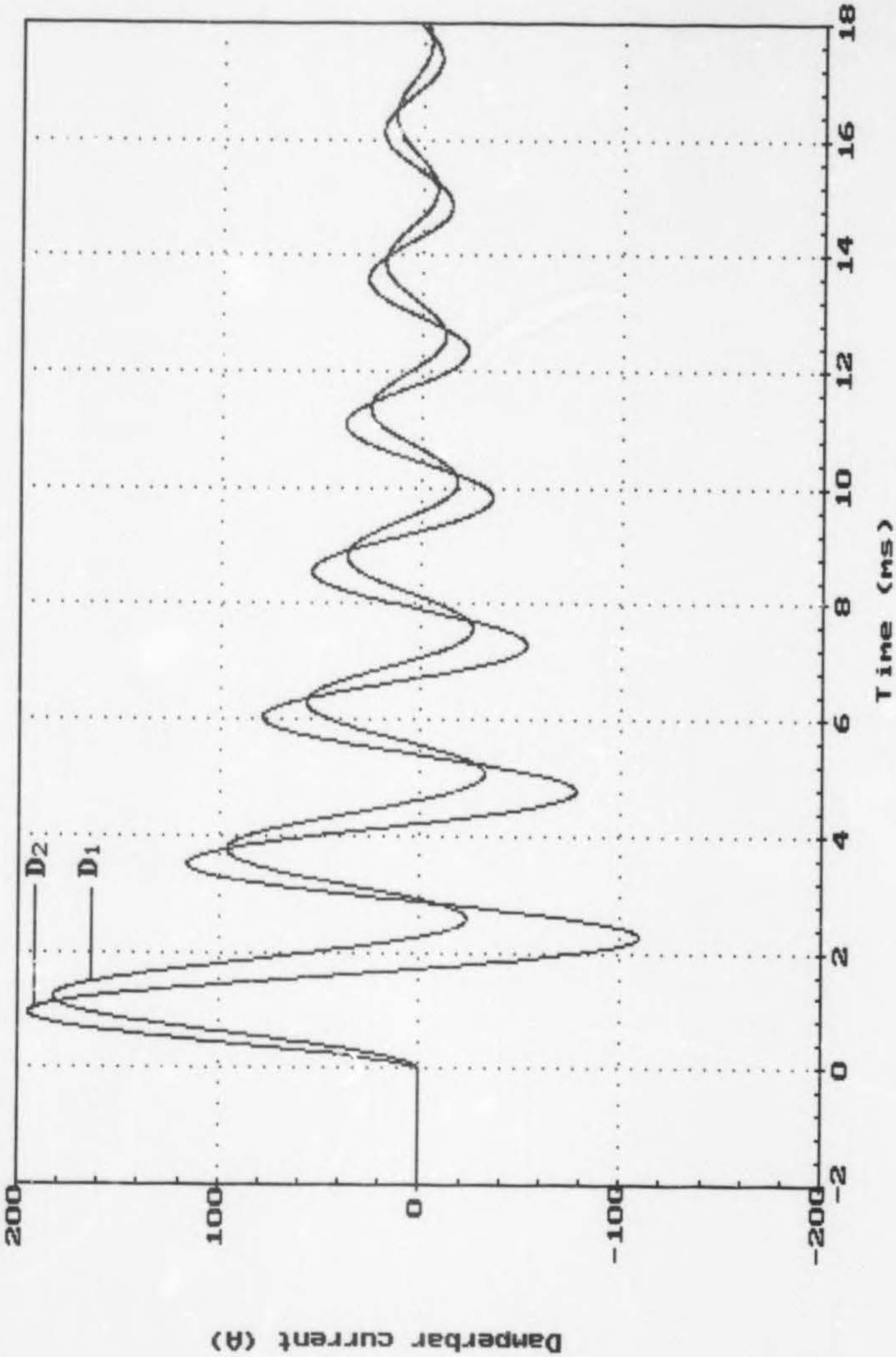

Figure 4.3: D-axis damper circuit currents after short-circuit

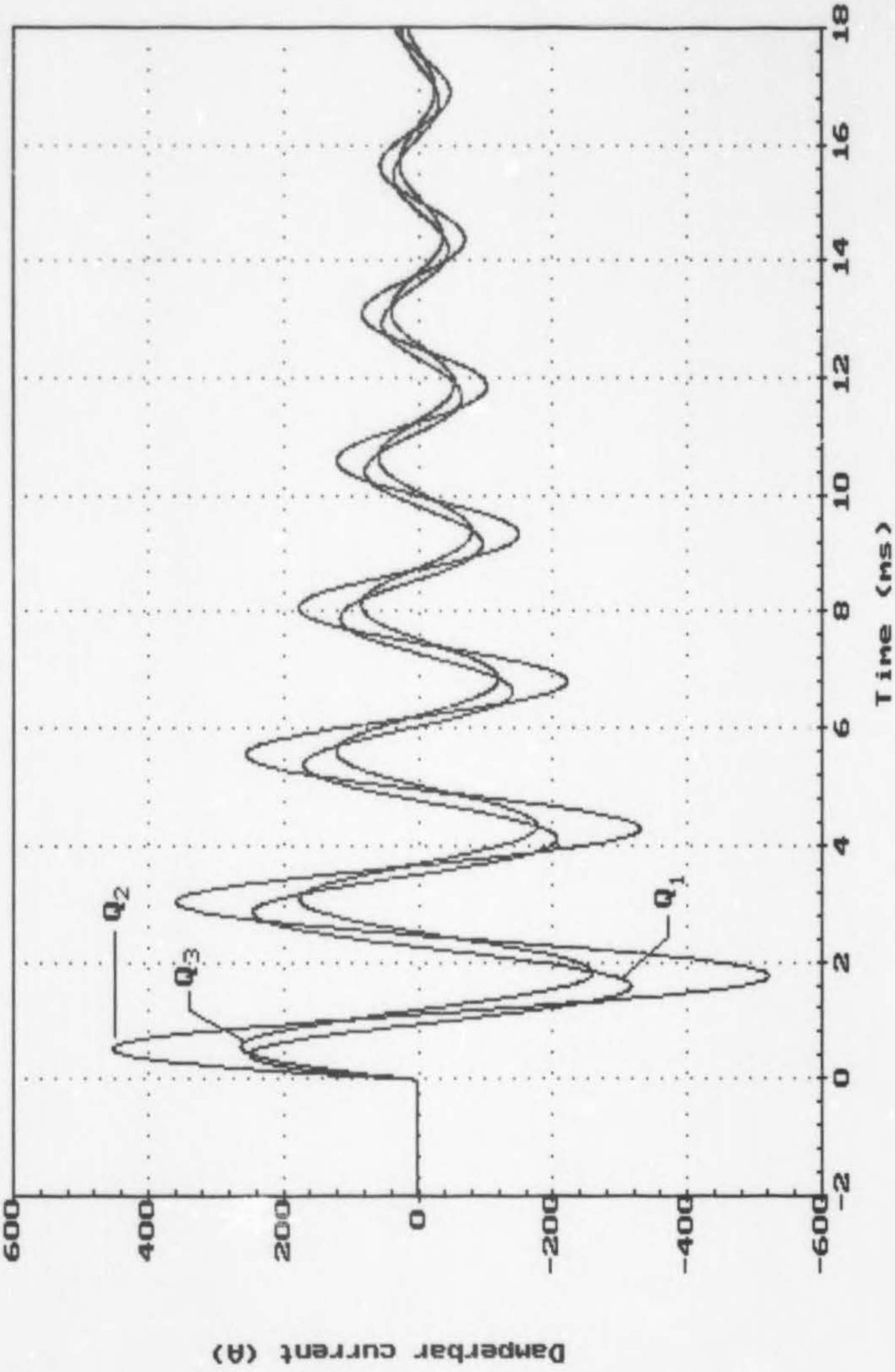

Figure 4.4: Q-axis damper circuit currents after short-circuit

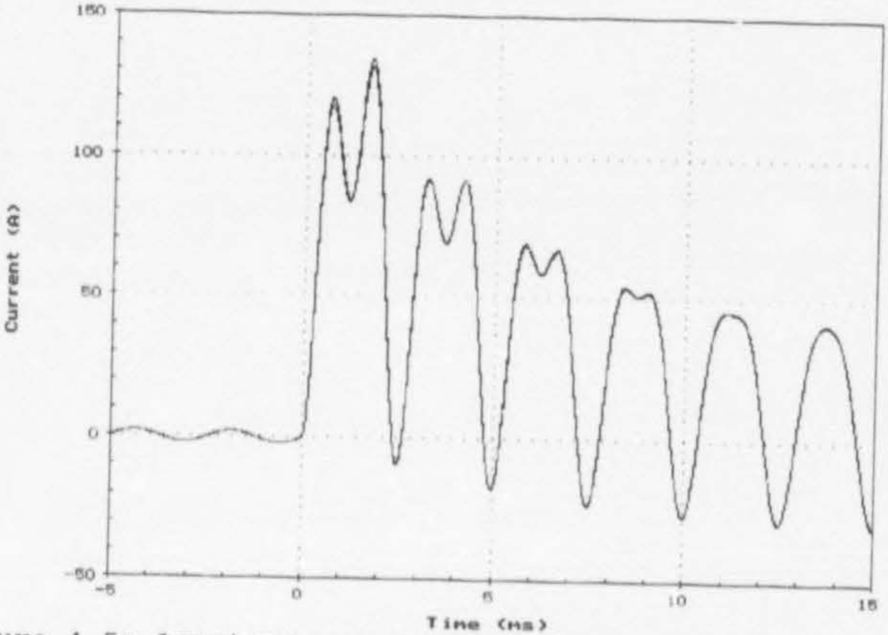

Figure 4.5: Armature currents of first generator after short-circuit

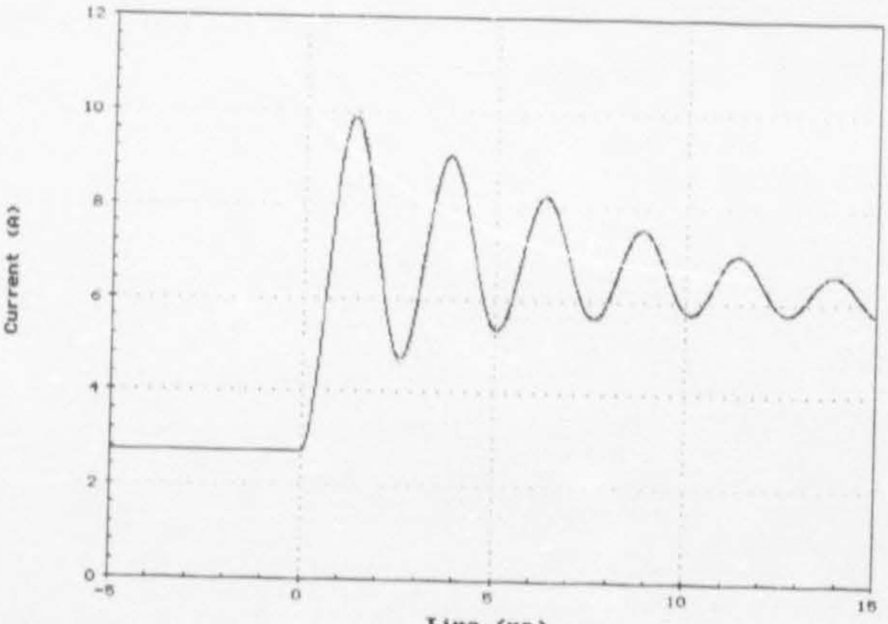

Figure 4.6: Field currents of first generator after short-circuit

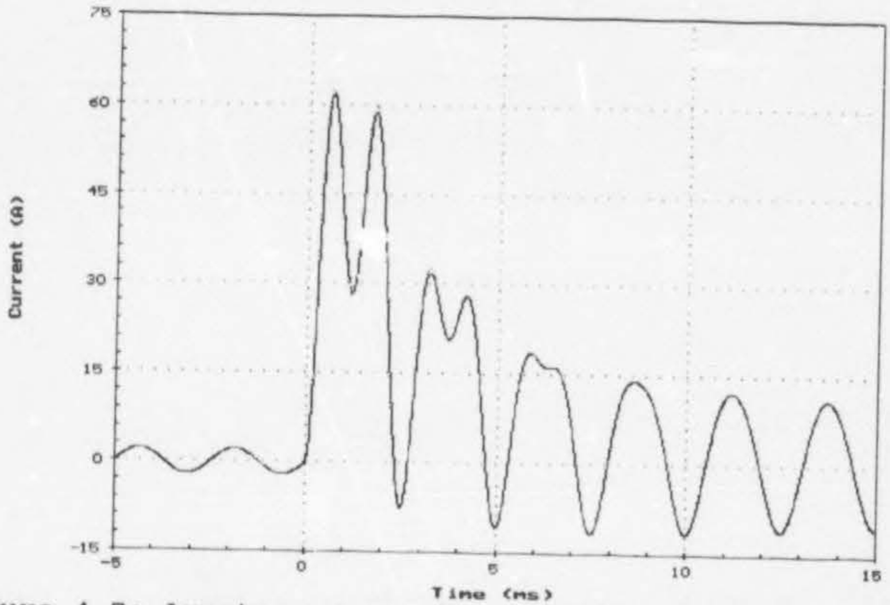

Figure 4.7: Armature currents of second generator after short-circuit

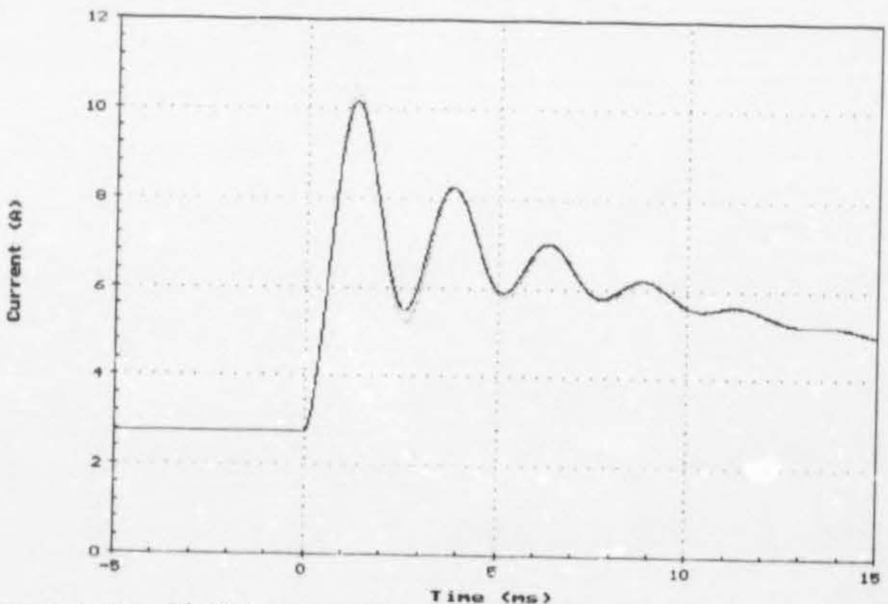

Figure 4.8: Field currents of second generator after short-circuit

4.5 CONCLUSIONS

As predicted, the sub-transient response of the reduced matrix differ slightly form the original response. For a terminal short-circuit, all the effective reactances are d-axis quantities. As described earlier, the d-axis damper bars force the stator flux into the airgap immediately after the fault. This creates the leakage reactance which determine the initial swing of the stator current. The difference between the actual waveshapes and the assumed waveshapes in this region is the reason for this difference in dynamic response of the currents. After the decay of the sub-transient component, the assumed waveshapes differ very little from the real waveshapes and the response of the reduced matrix is fairly accurate.

The error between the original and the reduced matrices is less than 5 % for both the armature and field currents. This method of reducing the matrix to an equivalent matrix thus gives a good representation of the response of the machine under the worst conditions. For conditions where the change in load is not so drastic, as in no-load to full-load, the effect of the sub-transient component does not play such a big role, and the response of the reduced matrix will be more accurate than with the short-circuit.

This is thus a good method of relating the original machine to an equivalent machine which is the same as the standard textbook unified machine theory.

5. SUMMARY OF CONCLUSIONS

After the impedance of the separate coils in the machine have been calculated, the impedance matrix of the machine can be derived from these values with the use of a coupling matrix. The thesis shows a way of deriving a coupling matrix between the currents of the single coils and the current matrix of the complete machine. From this relationship, the impedance matrix for the complete machine can be derived. In this case, the impedance matrix is a 9x9 element matrix.

This 9x9 impedance matrix is then used to transform the three-phase machine with a stationary reference frame to a two-phase machine with a rotating reference frame. The same principles used by Jones [1] were used here. This leads to a impedance matrix in which all the values are constants and not functions of time or rotor position.

The transformed matrix is then rewritten in such a way that the response of the currents due to a change in load conditions, can be calculated with numerical integration techniques. With the method used, different load conditions can be simulated. Saturation, however, is not taken into account in this simulation. Humpage [31] describes a way in which the inductance values are varied according to the magnetic saturation in the machine. By including a separate iteration to calculate the magnetic saturation, and changing the impedance values accordingly, saturation can be taken into account. This will then give a more accurate representation of the machine.

The simulation method is then used to find a method of relating the machine under consideration to the standard textbook theory. The assumption that the damper currents have the same waveshape proved to be good. In the region after the decay of the sub-transient component, the correlation between the two matrices are excellent. During the decay of the sub-transient component, the method is less accurate, but the error is still less than 5 % which is very good. If a better algorithm can be found to calculate an equivalent current, the method might give even better results. A perfect match however, will never be possible, because when one reduces the number of damper circuits, the complexity of the system is reduced and a certain amount of information is lost.

Another method to reduce the impedance matrix, is to consider the damper currents to have the same waveshape in time, with a sinusoidal distrubution in amplitude. This is the approach taken by most machine manufacturers. The sinusoidal distribution method can be applied to the D-axis currents, but does not correlate with the Q-axis currents. In this case the approach taken in the thesis seems to be a better one. Futher verification of the method can be done by doing tests on practical machines and correlate that with simulated results.

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7. APPENDICES

7.1 APPENDIX 1: LIST OF SYMBOLS

θ	Rotor position
λ	Flux linkage
μ_o	Permeability of air
ϕ	Position in airgap
Φ	Flux
B	Flux density
e	Instantaneous voltage
F	Voltage factor
$F(\phi)$	MMF at specific position in airgap
i	Instantaneous current
I	Current or current matrix
K	Constant for currents
L	Inductance
$L(\theta)$	Inductance that varies with rotor position
l_ϕ	Airgap length at specific position in airgap
p	Number of poles/derivative
P	Power dissipated
P_ϕ	Permeance per unit area
R	Resistance
V	Voltage or voltage matrix
W	Energy stored
Z	Impedance or impedance matrix

7.1.1 Subscripts

Q	Q-axis damper
D	D-axis damper
θ	Value that changes with rotor position
ϕ	Value that is a function of air-gap position

7.1.2 Sub-subscripts

1	First circuit
2	Second circuit
3	Third circuit
11	Influence of circuit on itself
12	Influence of one circuit on another
e	Equivalent value
T	Total

7.1.3 Superscripts

T	Transponent

7.2 APPENDIX 2: MATHEMATICAL DERIVATION OF C1-TRANSFORMATION

The C_1 transformation replaces the actual three-phase machine with its two-phase equivalent. This brings about a remarkable simplification of the voltage equations of the machine. The C_1 transformation does not eliminate θ and the equations are thus still functions of time after its application.

The simplest way to derive the C_1 transformation from physical considerations, is to consider the magnetomotive force distributions inside the machine. Figure 7.1 shows the axes of the MMF vectors of the three-phase winding, as well as the axes of the MMF vectors of the equivalent two-phase windings.

Figure 7.1: Three-phase and equivalent two-phase MMF's

Assume that there is no connection to the star-point of the machine. Then the relationship

$$i_r + i_y + i_b = 0$$

between the three instantaneous currents is satisfied. Assume that the number of turns is N_3 and N_2 for the three-phase and two-phase machines respectively. The resultant of the two sets of m.m.f.s should be the same and from figure 7.1 one gets

$$N_2 i_\alpha = N_3 (i_r - \tfrac{1}{2} i_y - \tfrac{1}{2} i_b)$$
$$N_2 i_\beta = N_3 \tfrac{\sqrt{3}}{2} (i_y - i_b)$$

Writing the ratio $N_3/N_2 = b$ the equation can be written as

$$i_\alpha = b(i_r - \tfrac{1}{2} i_y - \tfrac{1}{2} i_b)$$
$$i_\beta = b\tfrac{\sqrt{3}}{2} (i_y - i_b)$$

The transformation matrix must be orthogonal and the value of b will be determined according to this. The current in the fourth wire which is connected to the star point, cannot be simulated by the two-phase machine and its representation requires a separate independent circuit. The new current will be denoted by i_o. From the fact that it should be zero when the fourth wire is missing, it follows that it should be a constant multiple of the sum of the three currents i_r, i_y and i_b. The factor will be chosen as bc where the value of bc will be determined from the orthogonal condition. The three equations of the transformation can now be written as

$$i_0 = bc(i_r + i_y + i_b)$$
$$i_\alpha = b(i_r - \tfrac{1}{2} i_y - \tfrac{1}{2} i_b)$$
$$i_\beta = b\tfrac{\sqrt{3}}{2} (i_y - i_b)$$

In matrix form the transformation equation can be written as

$$\begin{bmatrix} i_0 \\ i_\alpha \\ i_\beta \end{bmatrix} = b \begin{bmatrix} c & c & c \\ 1 & -\frac{1}{2} & -\frac{1}{2} \\ & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} * \begin{bmatrix} i_x \\ i_y \\ i_y \end{bmatrix}$$

The requirements for orthogonality is

$$CC_t^* = C_t^* C = 1$$

For the transformation matrix C_1 the equation will thus be

$$C_t^* C = b \begin{bmatrix} c & c & c \\ 1 & -\frac{1}{2} & -\frac{1}{2} \\ & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} * b \begin{bmatrix} c & 1 & \\ c & -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ c & -\frac{1}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix}$$

$$= b^2 \begin{bmatrix} 3c^2 & & \\ & \frac{3}{2} & \\ & & \frac{3}{2} \end{bmatrix}$$

The answer will only be unity provided that

$$3b^2c^2 = 1 \quad \text{as well as} \quad \tfrac{3}{2}b^2 = 1$$

from which

$$b = \sqrt{\tfrac{2}{3}} \quad ; \quad c = \tfrac{1}{\sqrt{2}}$$

The orthogonal form of C_1 is thus

$$C_1 = \sqrt{\tfrac{2}{3}}\;\begin{array}{c} \\ r \\ y \\ b \end{array}\!\!\begin{array}{c} \begin{array}{ccc} \quad 0 \quad & \quad \alpha \quad & \quad \beta \quad \end{array} \\ \left[\begin{array}{ccc} \frac{1}{\sqrt{2}} & 1 & \\ \frac{1}{\sqrt{2}} & -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{1}{\sqrt{2}} & -\frac{1}{2} & -\frac{\sqrt{3}}{2} \end{array}\right] \end{array}$$

7.3 APPENDIX 3: MATHEMATICAL DERIVATION OF C2-TRANSFORMATION

The commutator transformation, C_2, is a solution to the purely mathematical problem of finding a way to eliminate θ in the impedance matrix. C_2 must therefore itself include θ which makes it a live transformation.

Figure 7.2 shows the stationary (α and β) as well as the transformed rotating two-phase (d and q) MMF vectors.

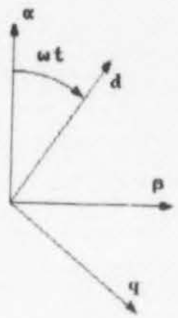

Figure 7.2: Stationary and rotating two-phase MMF's

From the figure C_2 can be written as

$$C_2 = \begin{array}{c} \\ 0 \\ \alpha \\ \beta \end{array} \begin{array}{c} \begin{array}{ccc} 0 & \quad q & \quad d \end{array} \\ \begin{bmatrix} 1 & & \\ & -\sin(\omega t) & \cos(\omega t) \\ & \cos(\omega t) & \sin(\omega t) \end{bmatrix} \end{array}$$

For a synchronous machine rotating at synchronous speed, the rotor angle θ is equal to ωt, and the ωt term can be replaced by θ. This gives the equation used in chapter 2.

7.4 APPENDIX 4: RUNGE-KUTTA NUMERICAL INTEGRATION

The Runge-Kutta (order four) algorithm can be described as follows:

To approximate the solution to the initial-value problem

$$x' = f(t,x), \quad a \le t \le b, \quad x(a) = \alpha,$$

select a positive integer N.

Step 1: Set $h = \frac{(b-a)}{N}$, $t_0 = a$, $w_0 = \alpha$

Step 2: Set $i=0$.

Step 3: Set

$$F1 = h*f(t_i, w_i),$$

$$F2 = h*f(t_i + \frac{h}{2}, w_i + \tfrac{1}{2}F1),$$

$$F3 = h*f(t_i + \frac{h}{2}, w_i + \tfrac{1}{2}F2),$$

$$F4 = h*f(t_i + h, w_i + F3),$$

$$w_{i+1} = w_i + \tfrac{1}{6}(F1 + 2F2 + 2F3 + F4),$$

$$t_{i+1} = a + (i+1)h.$$

Step 4: If $i = N$, goto *Step* 6.

Step 5: $i = i+1$; goto *Step* 3

Step 6: Stop.

After Step 6 the procedure is complete and w_i approximates $x(t_i)$, for each i = 1, 2, ..., N.

This method can be generalized as follows for an mth-order system. Let an integer N>0 be chosen and set h=(b-a)/N. Partition the interval [a,b] into N sub-intervals with the mesh points

$$t_j = a + jh \qquad \text{for each } j=0,\ 1,\ \ldots,\ N.$$

Use the notation w_{ij} to denote an approximation to $x_i(t_j)$ for each $j = 0, 1, \ldots, N$, and $i = 1, 2, \ldots, m$. For the initial conditions set

$$w_{i,o} = \alpha_i \quad \text{for each } i = 1,\ 2,\ \ldots,\ m.$$

The values of w_{ij} can now be calculated with the following set of equations.

$$F1_i = h*f_i(t_j,\ w_{1,j},\ w_{2,j}, \ldots, w_{m,j}) \quad for\ i=1,2,\ldots,m;$$

$$F2_i = h*f_i(t_j+\tfrac{h}{2},\ w_{1,j}+\tfrac{1}{2}F1_1, \ldots,\ w_{m,j}+\tfrac{1}{2}F1_m) \quad for\ i=1,2,\ldots,m;$$

$$F3_i = h*f_i(t_j+\tfrac{h}{2},\ w_{1,j}+\tfrac{1}{2}F2_1, \ldots,\ w_{m,j}+\tfrac{1}{2}F2_m) \quad for\ i=1,2,\ldots m;$$

$$F4_i = h*f_i(t_j+h,\ w_{1,j}+F3_1, \ldots,\ w_{m,j}+F3_m) \quad for\ i=1,2,\ldots m;$$

now

$$w_{i,j+1} = w_{i,j} + \tfrac{1}{6}[F1_i + 2F2_i + 2F3_i + F4_i] \quad for\ i=1,2,\ldots,m.$$

This set of equations was programmed with Turbo Pascal to solve the system of first-order differential equations of the currents in the machine.

7.5 APPENDIX 5: RUNGE-KUTTA ALGORITHM IMPLEMENTED WITH PASCAL

This program is a unit written for Turbo Pascal version 5 to solve the first-order differential equation of an electrical machine. The input must be in the format specified in the program. The answers is written to a file specified by Outfile which is a DOS text file.

```
Unit Initval;
{--------------------------------------------------------------------------}
{-                                                                        -}
{-  This unit includes routines that was taken from the                   -}
{-     Turbo Pascal Numerical Methods Toolbox                             -}
{-     Copyright (c) 1986, 87 by Borland International, Inc.              -}
{-                                                                        -}
{-  This unit provides procedures for solving initial value problems      -}
{-  for systems of coupled first-order ordinary differential equations    -}
{-  in the form                                                           -}
{-                      d          -1            -1                       -}
{-                      --[I] = [L]  * [V]  -  [L]  * [R] * [I]           -}
{-                      dt                                                -}
{--------------------------------------------------------------------------}

interface

{-------------------------------------------------------------}
{- Global variables that must be used by the program using   -}
{- this unit.                                                -}
{-------------------------------------------------------------}
const
  IVALNearlyZero = 1E-015;     { All values smaller = 0   }
  IVALArraySize  = 8;          { Maximum size of vectors  }
  MaxFuncs       = 8;          { Maximum number of user
                                 defined functions        }

type
  IVALvector  = array [0..IVALArraySize] of double;
  Matrix      = array [1..8,1..8] of double;

var
  RLmat                  : Matrix;       { R*Linv matrix     }
  InvLmat                : Matrix;       { Inverse L matrix  }
  V field                : double;       { Field voltage     }
  Spacing, HalfSpacing   : double;       { Time steps        }
  Index                  : integer;      { Counter           }
  Term                   : integer;      { Counter           }
  F1                     : IVALvector;   { General fourth    }
  F2                     : IVALvector;   { order Runge-Kutta }
  F3                     : IVALvector;   { formulas.         }
  F4                     : IVALvector;   {                   }
  CurrentValues          : IVALvector;   { Present values    }
  TempValues             : IVALvector;   { Temporary values  }

procedure InitialConditionSystem(NumEquations   : integer;
                                 LowerLimit     : double;
                                 UpperLimit     : double;
                                 NumIntervals   : integer;
                             var InitialValues  : IVALvector;
                             var outfile        : text;
                             var Error          : byte);
{--------------------------------------------------------------------------}
```

```
{-                                                                    -}
{- Input:  NumEquations, LowerLimit, UpperLimit, InitialValues,       -}
{-         NumIntervals                                               -}
{- Output:  SolutionValues, Error                                     -}
{-                                                                    -}
{- Purpose:  This procedure solves m 1st-order ordinary differential  -}
{-           equations with specified initial conditions.  The        -}
{-           generalized Runge-Kutta method solves for the m parameters -}
{-           in a given interval.                                     -}
{-                                                                    -}
{- User-defined Functions: IVALTargerF(No     : integer;              -}
{-                                     values : TNvector) : double;   -}
{-                                                                    -}
{- User-defined Types: TNvector = array[0..TNRowSize] of real;        -}
{-                     TNmatrix = array[0..TNColumnSize] of TNvector  -}
{-                                                                    -}
{- Global Variables: NumEquations : integer;   Order & number of eqs. -}
{-                   LowerLimit : real;        Lower limit of integration -}
{-                   UpperLimit : real;        Upper limit of integration -}
{-                   InitialValues : real;     Initial values of      -}
{-                                             The m parameters       -}
{-                   NumIntervals : integer;   Number of subintervals -}
{-                   Error : byte;             Flags if something goes -}
{-                                             wrong                  -}
{-                                                                    -}
{-            Errors:  0: No errors                                   -}
{-                     1: NumEquations < 1                            -}
{-                     2: LowerLimit = UpperLimit                     -}
{-                                                                    -}
{----------------------------------------------------------------------}

implementation

{-----------------------------------------------------------}
{- User defined functions - found from the current equation -}
{-----------------------------------------------------------}
function IVALTargetF( Fno : integer;
                      V   : IVALvector) : double;
var tmp1,tmp2,tmp3,tmp4  : double;
begin
  tmp1 := InvLmat[Fno,3]*V field;
  tmp2 := - RLmat[Fno,1]*V[1] - RLmat[Fno,2]*V[2];
  tmp3 := - RLmat[Fno,3]*V[3] - RLmat[Fno,4]*V[4] - RLmat[Fno,5]*V[5];
  tmp4 :=  - RLmat[Fno,6]*V[6] - RLmat[Fno,7]*V[7] - RLmat[Fno,8]*V[8];
  IVALTargetF := tmp1 + tmp2 + tmp3 + tmp4;
end; { function IVALTargetF1 }


procedure InitialConditionSystem({NumEquations   : integer;
                                 LowerLimit      : double;
                                 UpperLimit      : double;
                                 NumIntervals    : integer;
                             var InitialValues   : IVALvector;
                             var outfile         : text;
                             var Error           : byte});

procedure TestAndInitialize(NumEquations    : integer;
                            LowerLimit      : double;
                            UpperLimit      : double;
                            NumIntervals    : integer;
                        var Spacing         : double;
                        var Error           : byte);
{-------------------------------------------------------------}
{- Input: NumEquations, LowerLimit, UpperLimit,               -}
{-        NumIntervals                                        -}
{- Output: Spacing                                            -}
{-                                                            -}
{- This procedure initializes the above variables. Spacing is -}
{- initialized to (UpperLimit - LowerLimit)/NumIntervals.     -}
{- NumEquations, and NumIntervals are checked for errors; they -}
{- must be greater than zero.                                 -}
{-------------------------------------------------------------}
```

```
begin
  Error := 0;
  if NumEquations < 1 then
    Error := 1;
  if LowerLimit = UpperLimit then
    Error := 2;
  if Error = 0 then
  begin
    Spacing := (UpperLimit - LowerLimit)/NumIntervals;
  end;
end; { procedure TestAndInitialize }


procedure Step(    Spacing       : double;
               var CurrentValues : IVALvector;
               var F             : IVALvector);
{------------------------------------------------------------}
{- Input: Spacing, CurrentValues                            -}
{- Output: F                                                -}
{-                                                          -}
{- This procedure performs one step in the Runge-Kutta      -}
{- four-step algorithm. By varying CurrentValues, this      -}
{- procedure can be used at all steps in the Runge-         -}
{- Kutta algorithm.                                         -}
{-                                                          -}
{- This procedure assumes that there are MaxFuncs           -}
{- coupled 1st-order equations.  The F's record             -}
{- information about the MaxFuncs (or fewer) independent    -}
{- variables.                                               -}
{------------------------------------------------------------}
var i : integer;
begin
  for i := 1 to MaxFuncs do
      F[i] := Spacing * IVALTargetF(i,CurrentValues);
end; { procedure Step }


Procedure WriteValuesToFile(CurrentValues : IVALvector;
                            NumEquations  : integer    );
{------------------------------------------------------------}
{- This procedure write the values in CurrentValues to      -}
{- the file specified in outfile.  It is assumed that       -}
{- the file is already opene.                               -}
{------------------------------------------------------------}
var i : integer;
begin
  for i := 1 to NumEquations do
    write(outfile,CurrentValues[i]:12:6);
  writeln(outfile);
end;



{MAIN PROGRAM}
begin
  TestAndInitialize(NumEquations, LowerLimit, UpperLimit,
                    NumIntervals, Spacing, Error);
  if error=0 then
   begin
     CurrentValues[0] := LowerLimit;
     CurrentValues := Initialvalues;
     HalfSpacing := Spacing / 2;
     for Index := 1 to NumIntervals do
     begin
       { First step - calculate F1  }
       Step(Spacing, CurrentValues, F1);
       TempValues[0] := CurrentValues[0] + HalfSpacing;
       for Term := 1 to NumEquations do
         TempValues[Term] := CurrentValues[Term] + 0.5*F1[Term];

       { 2nd step - calculate F2  }
       Step(Spacing, TempValues, F2);
       for Term := 1 to NumEquations do
         TempValues[Term] := CurrentValues[Term] + 0.5*F2[Term];
```

```
      { Third step - calculate F3  }
      Step(Spacing, TempValues, F3);
      TempValues[0] := CurrentValues[0] + Spacing;
      for Term := 1 to NumEquations do
        TempValues[Term] := CurrentValues[Term] + F3[Term];

      { Fourth step - calculate F4[1]; first equation  }
      Step(Spacing, TempValues, F4);

      { Combine F1, F2, F3, and F4 to get  }
      { the solution at this mesh point    }
      CurrentValues[0] := CurrentValues[0] + Spacing;
      for Term := 1 to NumEquations do
        CurrentValues[Term] := CurrentValues[Term] + (F1[Term] +
                              2*F2[Term] + 2*F3[Term] + F4[Term])/6;
      WriteValuesToFile(CurrentValues,NumEquations);
    end;  {For index}
    InitialValues := CurrentValues;
  end;  {if error=0}
end; { procedure InitialConditionSystem }

end. { InitVal }
```